Low-Complexity Multiple-Component Turbo Decoding Aided Hybrid ARQ

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***Abstract*—Previous research has focused on improving the throughput of Hybrid Automatic Repeat reQuest (HARQ) schemes. However, since turbo codes have been introduced into HARQ schemes, their complexity has increased owing to the iterative Bahl, Cocke, Jelinek and Raviv (BCJR) operations that are required following each retransmission. This paper explores the complexity of turbo HARQ schemes and proposes a new Early Stopping (ES) approach for iterative decoding based on Mutual Information (MI), which dynamically determines the appropriate number of BCJR operations to be performed following each Incremental-Redundancy (IR) transmission. We demonstrate that the proposed ES based Multiple-Component Turbo Code (MCTC) aided and systematic Twin-Component Turbo Code (TCTC) assisted HARQ schemes exhibit a 60% to 85% reduced complexity for SNRs below -2dB, without degrading the Packet Loss Ratio (PLR) and the throughput.**

I. INTRODUCTION

Hybrid Automatic Repeat reQuest (HARQ) has proved to be an essential error control technique in communication networks [1, 2]. This combines ARQ and Forward Error Correction (FEC) to achieve a vanishingly low Bit Error Ratio (BER), which cannot be achieved by either method alone. A particular focus of recent research into HARQ schemes has been that of improving their throughput. For this reason, combining the corrupted retransmitted replicas using turbo codes has received attention, as a benefit of their potential of near-capacity operation. Souza's HARQ scheme [3] aided by systematic Twin-Component Turbo Codes (TCTC) and Chase combining, as well as our previously proposed Multiple-Component Turbo Coded (MCTC) HARQ scheme [4] have been shown to achieve high throughputs. The receivers of both schemes perform an iterative decoding process after each retransmitted packet is received. These iterative Bahl, Cocke, Jelinek and Raviv (BCJR) operations continue until the Cyclic Redundancy Check (CRC) is satisfied or a predefined number of BCJR iterations is reached, whereupon another retransmission is requested and iterative decoding recommences.

However, the choice of the affordable number of BCJR iterations employed in these schemes significantly affects their performance. If the number is set too low, then unnecessary retransmissions may be requested and the throughput will suffer. If the number is set too high, then unnecessary BCJR operations will be performed and hence the complexity will be increased. In fact, we will demonstrate that different numbers of BCJR operations are appropriate at different stages of the HARQ receiver's operation. Since the appropriate number is difficult to predict in advance, an intelligent on-line Early Stopping (ES) approach is needed.

Many ES approaches have been proposed since turbo codes were invented. Most of these ES schemes [5–9] halt the iterative decoding process when the magnitude of Logarithmic Likelihood Ratios (LLRs) exceeds an appropriately chosen threshold. The earliest schemes quantify the cross entropy between the distributions of the *a posteriori* LLRs generated by the two BCJR decoders [5, 6]. The authors of [8] suggested estimating the mean of the LLRs' absolute values, while Li and Wu in [9] employed the cross correlation between the LLRs. In classic turbo codes operating without ARQ, these ES approaches determine the specific instant to curtail iterative decoding by considering the expected BER performance. However, in turbo HARQ schemes, the ES approach decides when to request a new incremental transmission, rather than increasing the number of BCJR operations for the current codeword, hence striking a tradeoff between the attainable throughput and the complexity imposed.

In this paper, we propose a new ES approach that is specifically designed for turbo HARQ, based on the Mutual Information (MI) improvement after each BCJR operation. This is based on the observation that the MI is expected to gradually improve as iterative decoding proceeds, as quantified by the EXIT charts [10]. We refine this approach in order to strike an attractive tradeoff between the achievable throughput versus the complexity imposed for different packet lengths. We apply our ES approach to the above mentioned TCTC and MCTC aided HARQ schemes, demonstrating that the complexity is significantly decreased, while maintaining similar throughputs to those presented in [3] and [4].

The rest of this paper is organized as follows. Section II details the schematic of our MCTC HARQ scheme and describes its encoding and decoding process. Section III describes and parameterizes our proposed ES approach. Then, the performance of the schemes operating with and without our ES approach is characterized in Section IV. Section V concludes the paper.

II. SYSTEM MODEL

The complexity of a tubo decoder may be quantified in terms of the total number of trellis states per bit [11], which is expressed as $Complexity = 2^m \cdot K$, where 'm' is the number of memory elements employed in the convolutional encoders' generator polynomial, 2^m is the number of states in the corresponding trellis diagram and 'K' is the total number of BCJR operations performed during the iterative decoding process. Since the complexity exponentially increases with 'm', the generator polynomials of $(2, 3)_{Octal}$ having the lowest

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possible memory length of '$m = 1$' is desirable. However, for the systematic TCTC aided HARQ scheme, both the Packet Loss Ratio (PLR) and throughput performance degrade severely when adopting the polynomials of $(2,3)_o$ [4], as we will demonstrate in Section IV. On the other hand, [12] has demonstrated that the polynomial pair of $(2,3)_o$ is desirable for MCTC, since it is the one that achieves the lowest BER and facilitates the closest possible operation to the Discrete-input Continuous-output Memoryless Channel's (DCMC) capacity. Therefore, in this paper, the MCTC is employed in the HARQ scheme in order to facilitate the employment of the low complexity generator polynomials $(2,3)_o$.

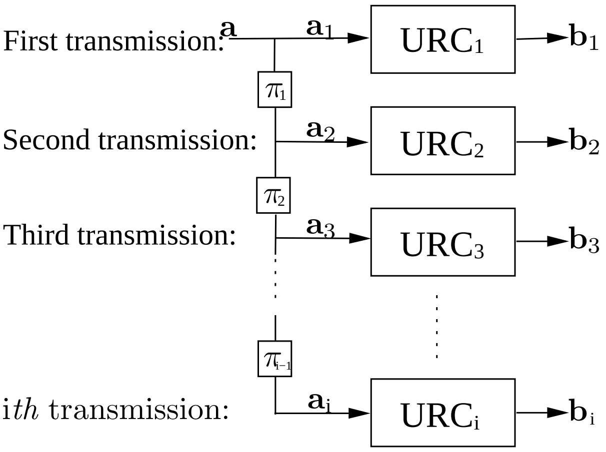

Fig. 1. The encoder structure of the HARQ scheme using MCTC.

The transmitter schematic of MCTC aided HARQ is illustrated in Figure 1, where the input bits $\mathbf{a}$ are obtained by appending CRC to the information bits. Then, the differently interleaved copies $\mathbf{a}_i$ are forwarded to recursive Unity Rate Code (URC) encoders. Note that these interleaving operations π_1,π_1,...,π_{i-1} may be realized by the same interleaver design, without degrading the system performance. This approach has the advantage of reducing the implementational complexity. The encoded bits $\mathbf{b}_i$ are sequentially transmitted at regular intervals, until a positive ACKnowledgement (ACK) message is received.

The receiver schematic is displayed in Figure 2 and the flow chart of the decoding process is illustrated in Figure 3. As seen in Figure 3, the receiver performs a single BCJR operation following the reception of the first encoded bit sequence $\tilde{\mathbf{b}}_1$, when the *a priori* LLRs $\tilde{\mathbf{a}}_1^a$ of the BCJR_1 have zero values. A hard decision is carried out on the basis of the output *a posteriori* LLRs $\tilde{\mathbf{a}}^p$ in order to obtain the decoded bits. If the CRC fails for the decoded bits, the receiver will wait until the transmitter times out and transmits the second encoded bit sequence $\tilde{\mathbf{b}}_2$. Turbo decoding commences after the reception of $\tilde{\mathbf{b}}_2$. Generally, whenever a set of encoded LLRs $\tilde{\mathbf{b}}_i$ ($i \geq 2$) is received during the ith Incremental Redundancy (IR)-transmission, the corresponding decoder BCJR_i is activated. It is immediately operated and becomes available for operation during the subsequent iterative decoding processes, yielding the i-component MCTC shown in Figure 2. Whenever a decoder BCJR_i is operated, the *a priori* uncoded LLRs $\tilde{\mathbf{a}}_i^a$ are obtained by interleaving and summing the extrinsic uncoded LLRs provided by the most recent operation of all other activated decoders. The decoder BCJR_i combines the *a priori* uncoded LLRs $\tilde{\mathbf{a}}_i^a$ with the *a priori* encoded LLRs $\tilde{\mathbf{b}}_i$, in order to obtain the extrinsic uncoded LLRs $\tilde{\mathbf{a}}_i^e$. Following this, the *a posteriori* uncoded LLRs $\tilde{\mathbf{a}}^p$ are obtained by interleaving and then summing the extrinsic uncoded LLRs provided by the most recent operation of all activated decoders. A hard decision is made for each bit on the basis of the *a posteriori* uncoded LLRs $\tilde{\mathbf{a}}^p$ and then the CRC check is performed. If the CRC fails and our proposed ES approach (represented by the blocks in the dashed polygon of Figure 3 and detailed in Section III) is not satisfied, the next BCJR decoder to be operated is selected as the one that was operated earliest, because it is the one that is most likely to provide the largest MI increment. As a result, a particular MI value can be reached using less BCJR operations, compared to other selection strategies. When the proposed ES approach is satisfied while the CRC still fails, a new IR-transmission is requested and the new round of turbo decoding is repeated until the corrected decoded bits are obtained, or the retry limit is reached.

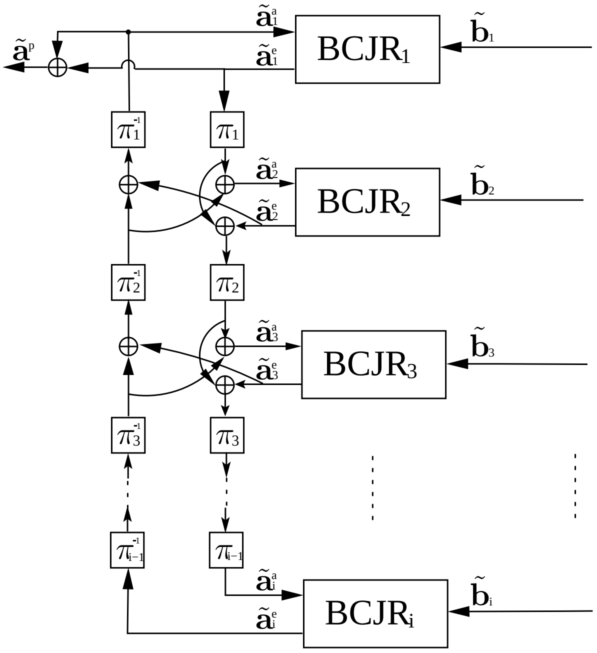

Fig. 2. The decoder structure of the HARQ scheme using MCTC after i IR-transmissions.

III. EARLY STOPPING APPROACH FOR THE MCTC AIDED HARQ SCHEME

The complexity metric of Section II suggests that once the $(2,3)_o$ polynomials have been adopted, the total number of BCJR operations 'K' performed during turbo HARQ decoding is the key factor that determines the complexity. Our ES approach aims for an efficient yet low-complexity design[1], having a minimum number of parameters in order to maximize the generality and applicability of the algorithm. There are

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[1] A complex strategy may outperform our efficient design at the cost of a disproportionately higher complexity. For example, our future work will determine whether to trigger the turbo decoding following each transmission, based on a prediction whether the MI of the current MCTC may get close to 1 according to a lookup table.

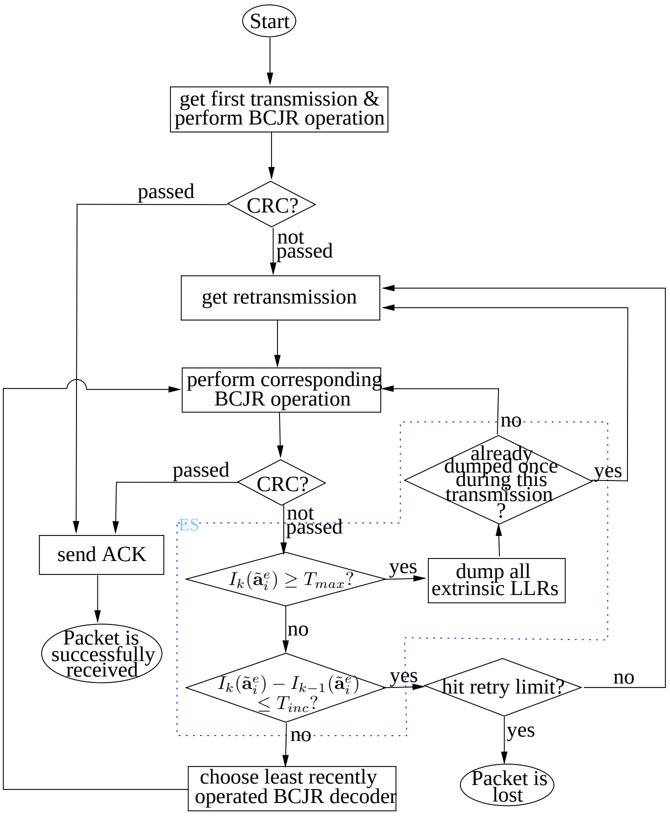


Fig. 3. The flow chart of the decoding process for the MCTC aided HARQ scheme.

two elements to our ES approach, both of which employ MI thresholds. One MI threshold is proposed to dynamically adjust the number of BCJR operations that are performed following each IR-transmission. We refer to this threshold as the ‘convergence threshold’. The second MI threshold is proposed to guarantee a zero PLR for the MCTC aided HARQ scheme relying on an infinite number of IR-transmissions. We refer to this threshold as the ‘dumping threshold’. These two thresholds are discussed in the following sections.

A. Analysis of the convergence and dumping thresholds

1) Convergence threshold: The MI of an LLR sequence gives an indication of confidence obtained for the corresponding hard decisions. According to [13], the MI can be estimated using the following equation:

$$I(\tilde{\mathbf{a}}) \approx 1 - \frac{1}{N}\sum_{j=1}^{N} H_b\left(\frac{e^{+|\tilde{a}_j|/2}}{e^{+|\tilde{a}_j|/2}+e^{-|\tilde{a}_j|/2}}\right) \quad , \qquad (1)$$

where N is the packet length, $|\tilde{a}_j|$ is the LLR’s absolute value at the bit position j and ‘H_b’ is the binary entropy function. Equation 1 maps the sequence of LLRs to a confidence metric confined to the range of $[0, 1]$, where 0 implies no confidence, 1 means absolute confidence. It is widely recognized that EXIT charts illustrate the MI transfer between the components of turbo codes. The bit-by-bit Monte-Carlo decoding trajectory steps within the EXIT chart tunnel may be treated as the MI increment after each iteration. The MI increment typically becomes smaller and smaller when the trajectory approaches the convergence point in the EXIT charts, where the turbo iterations should be stopped. Specifically, whenever a BCJR decoder is operated, the MI increment is expressed by $I_k(\tilde{\mathbf{a}}_i^e) - I_{k-1}(\tilde{\mathbf{a}}_i^e)$, where $I_k(\tilde{\mathbf{a}}_i^e)$ is the extrinsic MI obtained after the *kth* operation of BCJR_i and $I_0(\tilde{\mathbf{a}}_i^e)$ is assumed to be 0. Therefore, when the MI increment falls below a certain threshold, this may be considered as the stopping criterion, which we refer to as the convergence threshold T_{inc}.

The decoding iterations continue, as long as the MI increment is higher than T_{inc}, while an IR-transmission is requested when the increment drops below T_{inc}, as shown in Figure 3. Different values of T_{inc} result in different tradeoffs between the attainable throughput and the complexity imposed. Since the MI value varies from 0 to 1.0, the MI increment must also be confined to this range. When we have $T_{inc} = 0$, the receiver will only request a new IR-transmission when it is sure that it is not possible to satisfy the CRC without doing so, because for example a high number of BCJR operations have been performed and hence convergence to the wrong legitimate codeword is achieved. As a result, this approach minimizes the number of IR-transmissions at the cost of imposing a potentially high complexity. When T_{inc} assumes the maximum value of 1.0, the receiver will request new IR-transmissions as often as it can, in order to minimize the number of BCJR operations that are performed before the CRC is satisfied. As a result, this approach minimizes the complexity at the cost of having a low throughput. In Section III-B, we will observe the relationship between the attainable throughput and the complexity imposed by using different T_{inc} values for three typical packet lengths.

2) Dumping threshold: The MCTC aided HARQ scheme has an inherent problem, which prevents it from guaranteeing a PLR of zero, even when an infinite number of IR-transmissions is permitted. Specifically, turbo decoders are capable of converging to a legitimate bit-sequence for an MI of 1, but still output the wrong bit sequence. This occurs when the received soft-sequences are ‘closer’ to the incorrect bit sequence than to the correct sequence. This effect accounts for the error floor in the BER performance of turbo decoders at high Signal Noise Ratios (SNRs), where the EXIT chart tunnel is wide open. In the MCTC aided HARQ scheme, the decoding of the earlier few IR-transmissions may converge towards the wrong but legitimate bit sequence associated with a high confidence, like in a standard turbo code. In this case, the CRC fails and hence further IR-transmissions are requested. Unfortunately, the influence of these later IR-transmissions may become insufficiently decisive to guide the decoder away from the wrong bit sequence associated with MI ≈ 1 and towards the correct one, regardless of how many IR-transmissions are received. As mentioned in Section II, when an IR-transmission is received, the decoder first decodes this most recent IR-transmission. In a conventional regime following simple logic, the previously received codewords would provide *a priori* information for this most recent IR-transmission. However, in this scenario the previously received codewords may have a high extrinsic MI as a result of a comparatively high number of iterations, which is the reason for the relatively low influence for the most recent IR-transmissions. In other words, this approach activates the BCJR decoders corresponding to the earlier IR-transmissions more frequently than the other decoders. This lends these earlier IR-transmissions a higher influence, allowing them to persistently sway the iterative decoding process towards the incorrect uncoded bit sequence associated with MI ≈ 1. Therefore, repeated IR-transmissions will be requested again and again. If the IR-transmission retry limit is high, both the throughput and the complexity suffer significantly. Furthermore, the CRC may never be satisfied.

A dumping threshold T_{max} is employed to circumvent this problem as detailed below. When the MI of the extrinsic LLRs $\tilde{\mathbf{a}}^e$ obtained after some BCJR operation become higher than T_{max}, while the CRC has not been satisfied, all extrinsic LLRs are reset to zero and an improved iterative decoding procedure is activated, since the decoder is now in possession of potentially numerous received replicas of the original information. More explicitly, until this event the early replicas may have activated a higher number of iterations and hence may have driven the MI to high values, which lent them an increased influence over the more recent replicas. Recall that IR-transmission was only requested, when the CRC of the earlier ones failed. At this stage, we hence reactivate iterative decoding by exchanging extrinsic information amongst all replicas, lending them an equal influence, as a benefit of the above-mentioned LLR dump operation. For most cases, this re-initialized iterative process may lead to a better chance of successful decoding. In the exceptionally rare case that the MI obtained following a BCJR operation again becomes larger than T_{max} without satisfying the CRC, the receiver dumps all extrinsic LLRs once more and request a new IR-transmission. This process is repeated until the packet is correctly decoded. Assuming a perfectly reliable CRC, this allows a zero PLR to be attained, provided that the IR-transmission retry limit is sufficiently high.

As with T_{inc}, there is a tradeoff associated with the specific setting of T_{max}. If T_{max} assumes a large value close to 1, the receiver may require more IR-transmissions than necessary to ensure that the CRC will be satisfied, unless the extrinsic LLRs are dumped and the decoder is re-initialized. As a result, both the throughput and complexity suffer compared to adopting a smaller value of T_{max}. On the other hand, if T_{max} is set too small, the receiver may mistakenly think that the CRC will not be satisfied by performing more BCJR operations. As a result, 'genuine' LLRs will be dumped and more IR-transmissions will be requested, degrading both the throughput and complexity. Thus, the most appropriate value of T_{max} solves the wrong decision problem as well as approaching the optimum throughput at a low complexity. In Section III-B, we will determine the most desirable value of T_{max}.

B. Determining the stopping thresholds

In this section, we investigate the variation of the MCTC HARQ throughput and complexity with the value of stopping thresholds T_{inc} and T_{max}.

During our simulations, we determined the throughput and complexity that results for each combination of $T_{inc} \in \{1.0, 0.1, 0.01, 0.001, 0.0001\}$ and $T_{max} \in \{0.9, 0.99, 0.999\}$, at a range of channel SNRs and packet lengths. For the sake of observing the full influence of different stopping thresholds, an unlimited number of IR-transmissions was allowed in order to ensure that a PLR of zero was attained at all SNRs considered. Since the two stopping thresholds depend on the packet length, they were fitted to 100 bits, 1000 bits and 10000 bits in our simulations. In each simulation, a statistically significant number of packets was transmitted through a quasi-static Rayleigh fading channel using Binary Phase-Shift Keying (BPSK) modulation.

For the sake of conciseness, we detail only the process of selecting the preferred values of the convergence threshold T_{inc}, as provided in Table I for packets comprising 100, 1000 and 10000bits. The corresponding preferred values of the dumping threshold T_{max} of Table I were selected using an analogous process, which is not detailed in this paper. In summary, when the dumping threshold T_{max} is set to 0.99 for the 100-bit packets, and to 0.999 for both the 1000-bit and 10000-bit packet lengths, we obtain the highest throughput and the lowest complexity for the scenarios considered.

TABLE I
THE PREFERRED THRESHOLDS FOR DIFFERENT PACKET LENGTHS.

Packet Length	100 bits	1000 bits	10000 bits
T_{max}	0.99	0.999	0.999
T_{inc}	0.017	0.009	0.0032

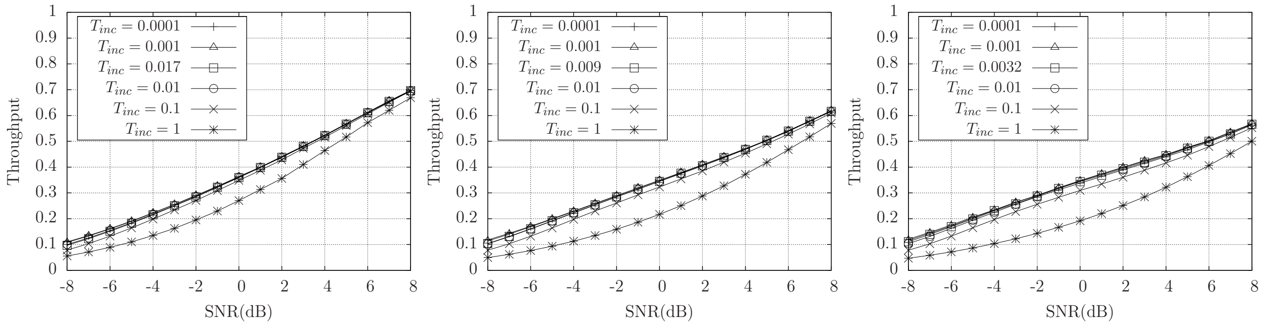
Figure 4 plots the throughput versus SNR for different convergence thresholds T_{inc} and packet lengths, when employing the corresponding dumping threshold values T_{max} of Table I. Here the throughput is defined as the ratio of the number of information bits delivered to the total number of transmitted bits. Figure 5 provides the corresponding complexity versus SNR curves, where the complexity metric was defined in Section II.

Figures 4 and 5 show that as the convergence threshold T_{inc} was increased, both the throughput and the complexity was reduced for all the packet lengths considered, as predicted in Section III-A. However, the throughput reduction was relatively modest, while the complexity was significantly reduced, as T_{inc} was increased from 0.0001 to 0.01. Since achieving a high throughput is the design target of HARQ schemes, we specify an average throughput reduction of 0.005 as the maximum loss that can be tolerated, when optimizing the corresponding T_{inc} values for all three packet lengths. We appropriately adjusted T_{inc} values in order to obey this throughput reduction limit. Quantitatively, we found that the preferred values were $T_{inc} = 0.017$ for the 100-bit, $T_{inc} = 0.009$ for the 1000-bit and $T_{inc} = 0.0032$ for the 10000-bit packet lengths, which yielded the lowest complexity. The curves of the throughput and complexity corresponding to these values were also included in Figures 4 and 5.

IV. PERFORMANCE RESULTS

In order to demonstrate the advantages of our ES approach, the PLR, throughput and complexity metrics are compared for Souza's systematic TCTC aided HARQ scheme [3] and our MCTC aided HARQ scheme.

The systematic TCTC of Souza's scheme employs the polynomials of $(17, 15)_o$ and uses a transmission limit of $r = 6$. During the $r = 6$ transmissions, the transmitted sequences are equivalent to $\mathbf{a}$, $\mathbf{b}_1$, $\mathbf{b}_2$, $\mathbf{a}$, $\mathbf{b}_1$ and $\mathbf{b}_2$ from Figure 1, where $\mathbf{a}$ is the vector of systematic bits, while $\mathbf{b}_1$ and $\mathbf{b}_2$ are the encoded

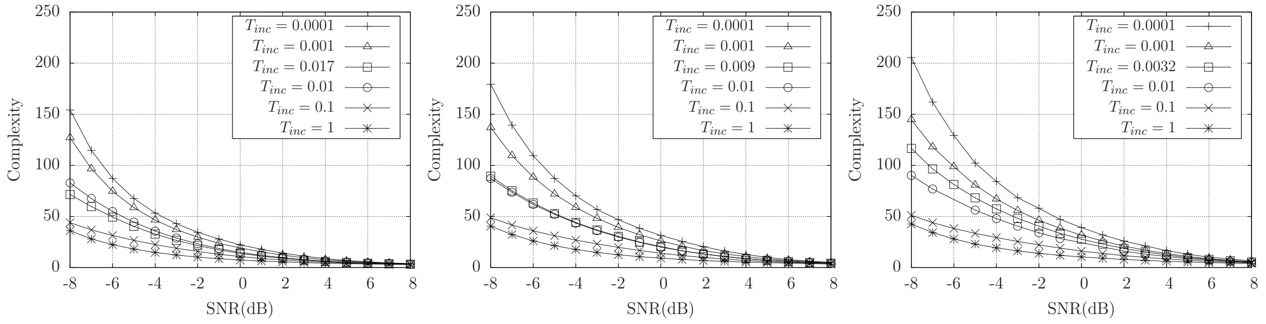


(a) 100bits, T_{max} = 0.99

(b) 1000bits, T_{max} = 0.999

(c) 10000bits, T_{max} = 0.999

Fig. 4. Throughput with different T_{inc} values for three different packet lengths. Transmissions take place over a quasi-static Rayleigh channel with an infinite number of retransmissions for a given packet, until it is correctly received.



(a) 100bits, T_{max} = 0.99

(b) 1000bits, T_{max} = 0.999

(c) 10000bits, T_{max} = 0.999

Fig. 5. Complexity with different T_{inc} values for three different packet lengths. Transmissions take place over a quasi-static Rayleigh channel with an infinite number of retransmissions for a given packet, until it is correctly received.

bits generated by a pair of URC encoders that are separated by an interleaver. This is in contrast to Souza's original scheme, in which $(\mathbf{a}, \mathbf{b}_1)$ are transmitted together during the first transmission, and the symbol energy E_s is doubled for the subsequent transmissions. However, we introduced this modification in order to maintain the same coding rate for the two HARQ schemes and to maintain the same symbol energy for each transmission. Furthermore, the receiver commences iterative decoding after the third transmission. The LLRs obtained from the incremental transmissions of two replicas of the same bit sequence are added together. For example, when the LLRs of the systematic bits $\mathbf{a}$ are soft-demodulated during the fourth transmission, they will be combined with the systematic LLRs obtained from the first transmission. Then, the twin-component turbo decoding recommences, preserving any extrinsic LLRs that were obtained following the third transmission. The same procedure continues for the fifth and sixth transmissions, if they are needed.

The packet length is set to a modest value of 1000 bits for both HARQ schemes. For the sake of fair comparison, the transmission limit is set to $r = 6$ for the MCTC aided HARQ scheme. Hence, some packet loss events are expected at low SNRs according to the flow chart of Figure 3. As suggested in [3], Souza's scheme performs a pre-defined number of 10 BCJR operations before stopping iterative decoding following each transmission. Correspondingly, our MCTC aided HARQ scheme uses the two stopping thresholds discussed in Section III. Given that the permitted throughput reduction is 0.005, according to Table I, the preferred dumping threshold T_{max} is 0.999 and the convergence threshold T_{inc} is 0.009 for the 1000-bit length packets. We also apply the proposed ES approach to Souza's HARQ scheme in order to evaluate our technique's generality. Likewise, the preferred convergence and dumping thresholds T_{inc} and T_{max} can be obtained using simulations similar to those described in Section III. The resultant preferred values for Souza's systematic TCTC HARQ scheme were found to be $T_{inc} = 0.1$ and $T_{max} = 0.999$ for the 1000-bit length, when the average throughput loss is limited to 0.005. Additionally, since Souza's original HARQ scheme adopts the memory length $m = 3$ generator polynomials of $(17, 15)_o$, we consider the effect of the code's memory length on the complexity by also employing the modified polynomials of $(2, 3)_o$ for Souza's scheme. For further exploring the ES advantages, our MCTC aided HARQ scheme was also configured to perform a pre-defined number of 10 BCJR operations following each transmission, in order to obtain an additional benchmarker that does not employ ES. All the transmissions are over a quasi-static Rayleigh fading channel using BPSK modulation.

Figure 6 illustrates the attainable PLR versus SNR performance, while Figure 7 shows the throughput versus SNR, where the throughput has the same definition as in Figure 4. It can be observed from these two figures that our ES-based MCTC aided HARQ scheme using the $m = 1$ polynomials of $(2, 3)_o$ succeeds in maintaining a similar PLR and throughput performance as that offered by Souza's more complex scheme using the $m = 3$ polynomials of $(17, 15)_o$, regardless of whether the proposed ES approach is adopted or not. However, both the PLR and throughput are significantly decreased if the polynomials are changed to $(2, 3)_o$ for Souza's scheme,

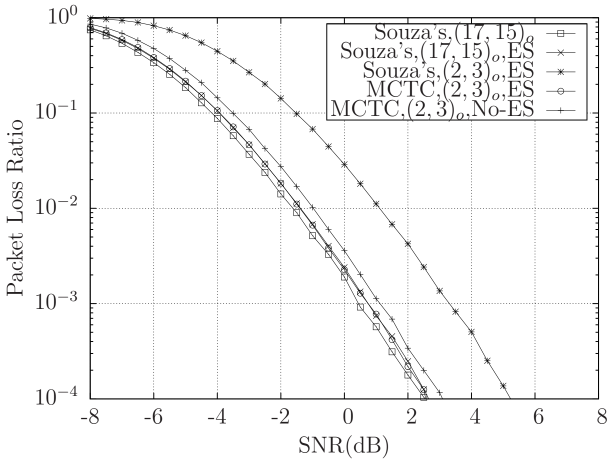


Fig. 6. The packet loss ratio versus SNR for transmission over a quasi-static Rayleigh channel, with the retransmission limit set to 6 and the packet length set to 1000 bits.

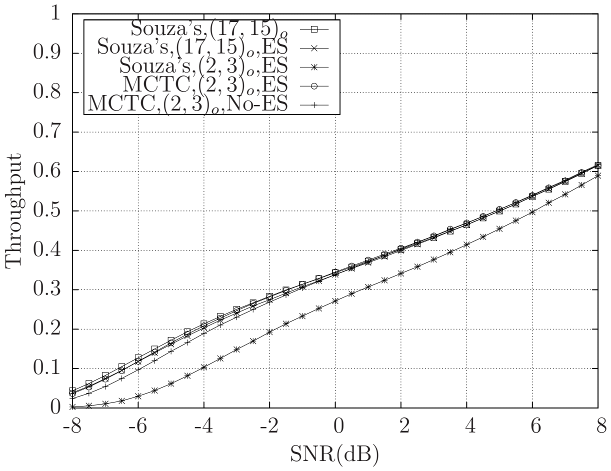


Fig. 7. Throughput versus SNR for transmission over a quasi-static Rayleigh channel, with the retransmission limit set to 6 and the packet length set to 1000 bits.

showing that this low-complexity polynomial pair is only appropriate for the proposed MCTC HARQ scheme.

Figure 8 shows the complexity benefits achieved by the proposed ES-based MCTC aided HARQ scheme, which has a significantly lower complexity than Souza's original systematic TCTC HARQ scheme relying on the $m = 3$ polynomials of $(17, 15)_o$. Specifically, for SNRs below -2dB, the complexity is reduced by 60% to 85%. At higher SNRs, the complexity is similar, because the CRC is typically satisfied for all schemes after a few BCJR operations. On the other hand, if Souza's systematic TCTC HARQ scheme using the $m = 3$ polynomials of $(17, 15)_o$ adopts our proposed ES approach, its complexity can be significantly reduced. However, its complexity still remains higher than that of our MCTC aided HARQ scheme, because it has to adopt those longer memory length polynomials. Although Souza's scheme using our proposed ES approach can achieve the lowest complexity for the $m = 1$ polynomials of $(2, 3)_o$, this is achieved at the cost of sacrificing the PLR and throughput performance, as demonstrated in Figures 4 and 5.

The PLR, throughput and complexity curves were recorded for the MCTC HARQ scheme without ES in Figures 6, 7 and 8, which further demonstrate how critical the careful

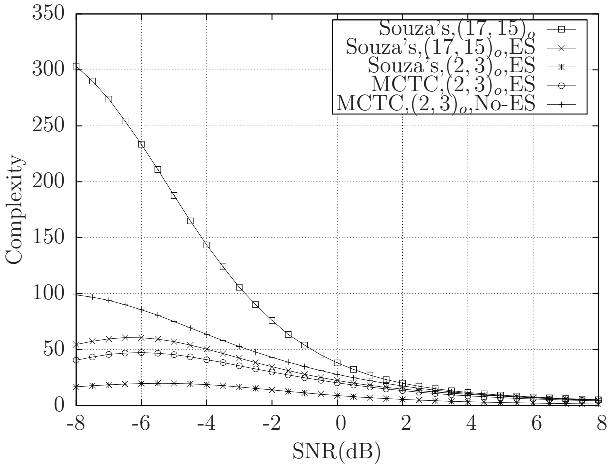


Fig. 8. Complexity versus SNR for transmission over a quasi-static Rayleigh channel, with the retransmission limit equal to 6 and the packet length set to 1000 bits.

employment of ES is for a turbo HARQ scheme. The PLR and throughput performances recorded for the No-ES MCTC HARQ scheme exhibit an insignificant degradation. Furthermore, the complexity more than doubles compared to that of the ES aided MCTC HARQ scheme for low SNRs, namely below -2dB. This illustrates that a fixed number of 10 BCJR operations is insufficient for reaching the maximum PLR and throughput that the MCTC HARQ scheme achieved for most situations, when the EXIT tunnel is open. By contrast, 10 BCJR operations appear to be excessive for most situations associated with closed EXIT tunnels, hence imposing an excessive complexity.

Finally, Table II compares the minimum number of variables that must be stored per bit of $\mathbf{a}$, for the MCTC HARQ scheme employing $r = 6$ BCJR decoders and for Souza's systematic TCTC HARQ scheme. Both schemes need to store one extrinsic LLR per BCJR decoder, namely 6 in the MCTC scheme and 2 in the TCTC scheme. Additionally, it is necessary to store one channel LLR per transmitted bit sequence, namely 6 in the MCTC scheme and 3 in the TCTC scheme. Furthermore, the authors of [14] demonstrated that at least 2^m alpha values per bit of $\mathbf{a}$ have to be stored between the forward and backward recursions of the log-BCJR algorithm, where alpha may be seen in Equation (5) of [15]. In summary, when assuming a retransmission limit of $r = 6$, the total memory requirements are 14 for the MCTC scheme and 13 for the TCTC schemes. Only one extra variable has to be required to be stored in memory per bit of $\mathbf{a}$ by the ES aided MCTC HARQ scheme, in order to achieve its remarkable performance improvement.

TABLE II

MEMORY REQUIREMENTS COMPARISON ASSUMING THAT THE RETRANSMISSION LIMIT IS $r = 6$.

(per bit of $\mathbf{a}$)	MCTC HARQ	TCTC HARQ
extrinsic LLR	6	2
channel LLR	6	3
alpha value	2	8

V. CONCLUSIONS

Classic turbo HARQ schemes [3] [4] may have a considerable complexity, since a pre-defined fixed number of turbo decoding iterations are performed following each transmission, without considering the channel conditions. In this paper, we have proposed an iterative decoding ES approach that maintains a high throughput and low PLR at a lower complexity. When comparing MCTC and systematic TCTC aided HARQ schemes, our simulation results show that the proposed ES approach is capable of decreasing the complexity imposed by as much as 85%. In our future work, we will focus our attention on the prediction of the stopping thresholds for different channel conditions in order to further reduce the complexity imposed.

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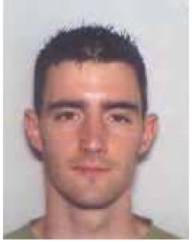

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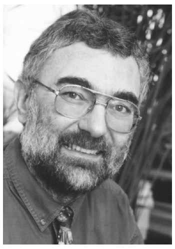

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