OPTIMIX:

OPTIMISATION OF MULTIMEDIA OVER WIRELESS IP LINKS VIA X-LAYER DESIGN

DELIVERABLE D2.2A

PRELIMINARY SCALABLE CHANNEL CODING AND SCALABLE MODULATION BEHAVIOUR AND MODEL

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Executive Summary

This document is the first one in a series of three describing the work carried out in Task 2.2, namely in (channel coding and modulation), both of which were given a similar emphasis.

The channel coding schemes were designed to satisfy stringent power and bandwidth constraints. More specifically, the proposed family of short block codes was designed for guaranteed convergence in soft-bit assisted iterative joint source and channel decoding, which assist iterative soft-bit source decoding to attain an infinitesimally low Bit Error Ratio (BER). Similarly, the class of protograph Low-density parity-check (LDPC) codes exhibits an attractive performance for transmission over a wide range of communication channels while substantially reducing the associated memory requirements without any BER degradation. Some classes of the LDPC code ensembles considered may even asymptotically approach the binary erasure channel’s (BEC) capacity. Some problems, however, arise when constructing finite length LDPC codes \((n, k)\) according the asymptotically optimal ensembles proposed in the open literature. Due to the reduced-complexity sub-optimal nature of iterative decoding at low signal-to-noise ratios (SNRs), typically there is a BER degradation in comparison to that of the same code under maximum likelihood (ML) decoding. Moreover, in the high-SNR, i.e. low BER region the associated BER performance curve typically exhibits an error floor imposed by the presence of small size stopping sets. In general, reducing the error floor under iterative decoding implies a sacrifice in terms of degrading it at low SNRs and vice versa. In this deliverable, the afore-mentioned issues are considered for the design of LDPC codes.

Radical novel modulation techniques are also considered in this document. One of these methods combines space-time block coding (STBC) based orthogonal transmit-diversity design with sphere packing (SP) modulation, which results in a useful reduction of the received SNR at a given BER. The crucial point at the receiver is that the concatenated detectors/demappers/decoders are soft-in/soft-out (SISO) decoders that accept and deliver probabilities or soft values, where the extrinsic information of the soft-output of one detector/demapper/decoder is passed on to the other detector/demapper/decoder to be used as a priori input. It is widely recognised that the choice of a specific bit-to-symbol mapping scheme or a beneficial constellation labelling is an influential factor, when designing iterative decoding based schemes exhibiting a high iteration gain. Moreover, the larger constellation size of a higher-dimensional space renders the bit-to-symbol mapping design more flexible as exemplified by using four-dimensional, rather than two-dimensional constellations. The optimisation of such mapping schemes is addressed in this document by employing the powerful reactive Tabu search algorithm.

Apart from the above-mentioned STBC-SP modulated single-user system we also considered multi-user systems. More specifically, we also investigate a novel singular value decomposition (SVD) assisted multiuser transmission (MUT) scheme in a multicell scenario. The SVD based scheme is capable of completely removing the co-channel interference, similarly to the classic zero forcing (ZF) based and block diagonalization (BD) aided MUT schemes. Two different power allocation schemes are investigated for SVD, ZF and BD based multicell transmissions. The SVD scheme achieves a suboptimal performance, but at a reduced complexity. Nonetheless, it always outperforms the ZF based scheme due to the joint reception of the transmitted symbols.

Our investigations evolve then further by quantifying the benefits of relaying and cooperation, which have attracted substantial research attention. In fact, the IEEE 802.16 working group has formed a special task group to include relay capabilities into the WiMAX standard. This task group is currently working towards completing the 802.16j Multihop Relay specification in order to improve the 802.16e-2005 system’s capabilities. The benefits of relaying are at least two-fold, firstly that of increasing the capacity, when the relays cooperate with base station in order to send the same data to the mobile station; secondly that of increasing the coverage area, when the relays route data arriving from the base stations, which cannot be received by remote mobile stations. Both modes are supported in the specification. In this document, we provide recommendations for the design of low-complexity scalable space time coding schemes for the implementation of the cooperative communication concept in the downlink of a wireless communication system.

Finally, the contributions of this task (Task 2.2, Channel coding and modulation) towards the OPTIMIX simulation chain are presented in terms of the planned integration of channel codes and modulation schemes. The design of channel encoders and decoders as well as modulators and demodulators is detailed along with all of their inputs, outputs and configuration parameters.
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1 Introduction

This document describes the current status of the channel coding module and the modulation module. Various novel channel coding and modulation techniques are studied and proposed. This document is structured as follows.

Chapter 2 describes our research activities on channel coding. In Section 2.1, we consider short block codes designed for guaranteed convergence in soft-bit assisted iterative joint source and channel decoding. We also design codes based on the LDPC approach of Section 2.2, whose excellent properties are well-known.

In Chapter 3, we present our recent research activities on attractive modulation techniques. We considered space-time block coding (STBC) in Section 3.1, where the transmitted signal space is optimised using sphere packing modulation. In Section 3.2, constellation labelling of bit-interleaved coded modulation assisted by iterative decoding (BICM-ID) is viewed as a quadratic index assignment problem and optimised using the powerful Reactive Tabu Search (RTS) algorithm. The application of singular value decomposition (SVD) assisted multiuser transmission in a multicell scenario is studied in Section 3.3. Two different power allocation schemes are investigated for SVD, ZF and BD based multicell transmission. It will be shown that the SVD scheme achieves a suboptimal performance, but at a reduced complexity. In Section 3.4, we provide recommendations for the design of a low-complexity scalable space time coding scheme for the implementation of the cooperative concept in the downlink of a wireless communication system. More precisely, we consider a pragmatic space-time coding approach based on the concatenation of convolutional codes and BPSK/QPSK modulation in order to design cooperative codes for relay networks, for which we derive the pairwise error probability, an asymptotic bound for frame error probability, and a design criterion to optimize both the achievable diversity and coding gain.

Finally, in Chapter 4, the blocks for channel coding and modulation are described as their OMNeT++ modules. Besides the specification of the interfaces, also some specific properties of the functionality are described.
2 Channel Coding

The first technique reported upon by this deliverable is the channel coding. Also known as the field of error correction coding [8], channel coding aim is to protect the information to be transmitted over a non-perfect channel, in order for it to be correctly received at the receiver side. This property can be achieved within limits established by Information Theory, the impassable limit being the capacity of the channel, above which no sure error-less decoding can be obtained.

In this section, we consider short block codes designed for guaranteed convergence in soft-bit assisted iterative joint source and channel decoding. We also deal with codes based on LDPC approach, whose excellent properties in terms of efficiency are well-known. Nevertheless, some more simple codes such as convolutional codes, Reed-Solomon codes, etc, will also be used within OPTIMIX project, if only as reference basis due to their wide usage in current standards.

2.1 Short Block Codes for Guaranteed Convergence in Soft-Bit Assisted Iterative Joint Source and Channel Decoding

2.1.1 Preliminaries

Short block codes designed for guaranteed convergence in soft-bit assisted iterative joint source and channel decoding are proposed, which assist iterative Soft-Bit Source Decoding (SBSD) to attain an infinitesimally low Bit Error Ratio (BER). SBSD was proposed by Fingscheidt and Vary [1] for improving the convergence of Iterative Source-Channel Decoding (ISCD) [2] by exploiting the natural residual redundancy of the source-coded bit-stream. However, when using sophisticated state-of-the-art coding techniques only modest residual redundancy is left in the source coded bit-stream, therefore we propose to intentionally impose additional redundancy on the source coded bit-stream with the aid of the novel class of Short Block Codes (SBCs) proposed.

2.1.2 The Algorithm

The conventional SBSD scheme derives the extrinsic information from the natural residual redundancy, which remains in the bit-stream after source encoding. More explicitly, the source-encoded bit-stream is partitioned into blocks of length $N$ symbol, each of which has a different probability of occurrence and will be termed as the information word encoded by the proposed SBC. We then characterise the redundancy of the source bit-stream with the aid of the non-uniform $M=2^N$-ary symbol probability distribution $P[s_i(\tau)]$, where $s_i(\tau)=[s_{i}(1), s_{i}(2), ... s_{i}(M)]$, with $k=1,2,...,N$ denoting the number of bits in each $M=2^N$-ary symbol. The details of the algorithm used for generating the extrinsic channel output information using SBSD for the zero-order Markov model can be found in [2]. Provided that the bits of an $M=2^N$-ary symbol may be considered independent of each other, the channels' output information generated for the $\tau$-th $N$-bit symbol is given by the product of each of the constituent single-bit probabilities as follows [3]:

$$P[\hat{y}_{(\tau,k,z)} \mid y_{(\tau,k,z)}] = \prod_{i=1}^{N} P[\hat{y}(i)_{(\tau,k,z)} \mid y(i)_{(\tau,k,z)}],$$

where $\hat{y}_{(\tau,k,z)} = [\hat{y}(1)_{(\tau,k,z)}, \hat{y}(2)_{(\tau,k,z)} ... \hat{y}(N)_{(\tau,k,z)}]$, is the received $N$-bit sequence. For each desired bit $y(\lambda)_{(\tau,k,z)}$, the extrinsic channel output information $P[y_{ext}^{(\tau,k,z)} \mid y_{(\tau,k,z)}]$ is expressed as:

$$P[y_{ext}^{(\tau,k,z)} \mid y_{(\tau,k,z)}] = \prod_{i=1,i\neq \lambda}^{N} P[\hat{y}(i)_{(\tau,k,z)} \mid y(i)_{(\tau,k,z)}].$$

Finally, the resultant extrinsic LLR value can be obtained for each bit by combining its channel output information and the a-priori knowledge of the corresponding $\tau$-th symbol as [1][2]

$$LLR \{y(\lambda)_{(\tau,k,z)}\} = \log \left\{ \frac{\sum_{y_{ext}^{(\tau,k,z)}} P[y_{ext}^{(\tau,k,z)} \mid y(\lambda)_{(\tau,k,z)} = +1] \cdot \prod_{i=1,i\neq \lambda}^{N} P[\hat{y}(i)_{(\tau,k,z)} \mid y(i)_{(\tau,k,z)}]}{\sum_{y_{ext}^{(\tau,k,z)}} P[y_{ext}^{(\tau,k,z)} \mid y(\lambda)_{(\tau,k,z)} = -1] \cdot \prod_{i=1,i\neq \lambda}^{N} P[\hat{y}(i)_{(\tau,k,z)} \mid y(i)_{(\tau,k,z)}]} \right\}.$$
Using efficient state-of-the-art encoders, only limited source redundancy is left in the source coded bit-stream, which typically results in modest system performance improvements beyond two decoding iterations. Hence carefully controlled redundancy is imposed by the proposed rate \( r = \frac{N}{(N+1)} \) SBCs to ensure that the resultant \( [N+1] \)-bit codewords exhibit a minimum Hamming distance of \( d_H = 2 \) between the \( M = 2^N \) number of legitimate \( N \)-bit source code words, which was shown to be the necessary and sufficient condition for achieving the highest possible source entropy denoted as

\[
H(X) = L_{SBSD}^{\text{exp}} = 1 \text{ bit,}
\]

provided that the input a-priori information of the SBSM is perfect, i.e. we have [4]

\[
H(X) = L_{SBSD}^{\text{apri}} = 1 \text{ bit.}
\]

According to our proposed SBC encoding procedure, first a redundant bit \( r_\tau \) is generated for the \( \tau \)-th \( M \)-ary source word by calculating the XOR function of its \( N \) constituent bits, according to

\[
r_\tau = [b^\tau(1) \oplus b^\tau(2) \ldots \oplus b^\tau(N)],
\]

where \( \oplus \) represents the XOR operation. The resultant redundant bit can be incorporated in any of the \( [N+1] \) different bit positions of the resultant SBC, in order to create \( [N+1] \) different SBC-encoded word combinations, as depicted in Table 1, each having a minimum Hamming distance of \( d_H = 2 \) from all the others.

<table>
<thead>
<tr>
<th>Symbols</th>
<th>( C_1 )</th>
<th>( C_2 )</th>
<th>\ldots</th>
<th>( C_{N+1} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( S_{(1)} )</td>
<td>( r_1 b_1 b_2 \ldots b_N )</td>
<td>( b_1 r_1 b_2 \ldots b_N )</td>
<td>\ldots</td>
<td>( b_1 b_2 \ldots b_N r_1 )</td>
</tr>
<tr>
<td>( S_{(2)} )</td>
<td>( r_2 b_1 b_2 \ldots b_N )</td>
<td>( b_1 r_2 b_2 \ldots b_N )</td>
<td>\ldots</td>
<td>( b_1 b_2 \ldots b_N r_2 )</td>
</tr>
<tr>
<td>( \vdots )</td>
<td>( \vdots )</td>
<td>( \vdots )</td>
<td>\ldots</td>
<td>( \vdots )</td>
</tr>
<tr>
<td>( S_{(N+1)} )</td>
<td>( r_{N+1} b_1 b_2 \ldots b_N )</td>
<td>( b_1 r_{N+1} b_2 \ldots b_N )</td>
<td>\ldots</td>
<td>( b_1 b_2 \ldots b_N r_{N+1} )</td>
</tr>
</tbody>
</table>

2.1.3 Design Example

The schematic of our proposed videophone arrangement used as our design example for quantifying the performance of various SBC schemes is shown in Figure 1. At the transmitter side the video sequence is compressed using the H.264 video codec and the video source bit-stream \( x_k \) is mapped or encoded into the bit-string \( x'_m \) using the specific bit-coding scheme of the SBC employed. Subsequently the output bit-string of the SBC is interleaved using the bit-interleaver \( \Pi \) of Figure 1, yielding the interleaved sequence \( x'_m \), which is then encoded by a Recursive Systematic Convolution (RSC) code having a specific code rate. The resultant bit-stream is QPSK modulated and transmitted over a correlated narrowband Rayleigh fading channel, associated with the normalised Doppler frequency of \( f_D = 0.01 \). The received signal is QPSK demodulated and the resultant soft-information is passed to the RSC decoder. The extrinsic information gleaned is then exchanged between the SBSM and RSC decoders of Figure 1, in order to attain the lowest possible BER.
2.1.4 Results

The achievable system performance was evaluated using the "Akiyo" video sequence [5] consisting of 45 (176 × 144)-pixel Quarter Common Intermediate Format (QCIF) frames and encoded using the H.264/AVC JM 13.2 reference video codec at 15 frames-per-second (fps) at the target bitrate of 64 kbps. Each QCIF frame was partitioned into 9 slices and each slice was composed of 11 Macro-Blocks (MBs) of a row of MB's within a QCIF frame. An intra-coded 'I' frame was inserted in the video sequence after every 45 frames, in order to curtail error propagation. Additionally, to control the effects of error propagation, we used intra-frame coded MB updates of three randomly dispersed MBs per frame. The coding parameters of the different SBC schemes are shown in Table 2. The overall code-rate of $R=1/2$ was maintained by adjusting the puncturing rate of the RSC in order to accommodate the different SBC rates of Table 2. The attainable BER performance using 2/3, 3/4, 4/5, 5/6, and 6/7 rate SBCs is shown in Figure 2. It is observed from Figure 3 that SBSD operating in conjunction with rate-1 SBC results in an inferior PSNR performance in comparison to all the other SBC schemes having a rate lower than 1, with an additional $E_b/N_0$ gain of 20 dB, relative to the rate-1 SBC, which in effect dispensed with the SBC.

### Table 2 - Code rates for different Error Protection schemes.

<table>
<thead>
<tr>
<th>SBC</th>
<th>Code Rate</th>
<th>RSC</th>
<th>SBC</th>
<th>Overall</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rate-1</td>
<td></td>
<td>1/2</td>
<td>1</td>
<td>1/2</td>
</tr>
<tr>
<td>Rate-2/3</td>
<td></td>
<td>3/4</td>
<td>2/3</td>
<td></td>
</tr>
<tr>
<td>Rate-3/4</td>
<td></td>
<td>2/3</td>
<td>3/4</td>
<td>1/2</td>
</tr>
<tr>
<td>Rate-4/5</td>
<td></td>
<td>5/8</td>
<td>4/5</td>
<td>1/2</td>
</tr>
<tr>
<td>Rate-5/6</td>
<td></td>
<td>3/5</td>
<td>5/6</td>
<td>1/2</td>
</tr>
<tr>
<td>Rate-6/7</td>
<td></td>
<td>7/12</td>
<td>6/7</td>
<td>1/2</td>
</tr>
</tbody>
</table>
Figure 2: BER performance of various error protection schemes.

Figure 3: PSNR-Y performance of various error protection schemes.
2.2 LDPC and Raptor codes for erasure channels: elements on code design and MBMS applications

2.2.1 Preliminaries

Low-density parity-check (LDPC) codes [10] exhibit an extraordinary performance under iterative decoding over a wide range of communication channels. Some classes of LDPC code ensembles under iterative decoding can even asymptotically approach with an arbitrarily small gap the binary erasure channel (BEC) capacity [11], [12], [13]. Some problems, however, arise when constructing finite length \((n, k)\) LDPC codes according to the so far proposed asymptotically optimal ensembles. Due to the sub-optimality of iterative decoding, at high error rates the performance curve, though usually good, denotes a coding gain loss with respect to that of the same code under maximum likelihood (ML) decoding. Moreover, at lower error rates the performance curve typically exhibits a high error floor caused by the presence of small size stopping sets. In general, lowering the error floor under iterative decoding implies a sacrifice in terms of coding gain high error rates.

Maximum likelihood (ML) decoding of a linear block code over the BEC is equivalent to solving the linear system

\[
x_{U} \cdot H_{U}^\top = x_{K} \cdot H_{K}^\top
\]

through a Gaussian elimination (GE) performed on the binary matrix \(H_{U}^{-1}\). In (1), \(x_{U}\) (resp. \(x_{K}\)) denotes the set of erased (resp. correctly received) encoded bits and \(H_{U}\) (resp. \(H_{K}\)) the submatrix composed of the corresponding columns of the parity-check matrix \(H\). The complexity of GE is in general cubic with respect to the dimension of the system, so that the overall complexity is \(O(n^3)\), where \(n\) is the codeword length. In general, for long block lengths ML decoding becomes impractical. However, in the specific case of LDPC codes it is indeed possible to perform ML decoding for longer codes (e.g., \(n\) up to thousands of symbols) taking advantage of the sparseness of the parity-check matrix. Efficient ways of implementing ML decoders for LDPC codes exploiting the parity-check matrix sparseness can be found, for example, in [24].

Failures of the iterative LDPC decoder over the BEC are due to stopping sets [14]. Since there exist sets of variable nodes (VNs) representing stopping sets for the iterative decoder, but not for the ML decoder, a decoding strategy more powerful than simple iterative decoding consists of performing iterative decoding at first and, upon an iterative decoder failure, employing the ML decoder to try to resolve the residual stopping set (for instance, see [15]). This hybrid IT/ML decoder achieves the same performance as an ML decoder. The performance curve obtained after the ML step achieves a higher coding gain than iterative decoding and a lower error floor, as the error floor under ML decoding depends solely on the distance spectrum\(^1\).

Recently, Raptor codes have been introduced in [17], and a subclass of Raptor codes has been included in the specification of the Multimedia Broadcast Multicast Service (MBMS) of the UMTS system [20]. For this subclass of Raptor codes, a low-complexity version of the ML erasure decoding is recommended. As explained later in this document, equation holds for Raptor codes as well, by properly updating with the constraint Raptor matrix (see Section 2.2.4.3).

2.2.2 Applications

The main application of erasure correcting codes in wireless communication systems is strictly related to the concept of packet loss recovery, as pointed out in [18].

Let us focus on the simple case of linear block codes. At the physical layer, a linear block code is usually employed to protect a frame by introducing redundancy. This redundancy is exploited on the receiver side to recover from errors introduced by the communication channel. An \((n, k)\) linear block code can be used also at higher layers of the protocol stack, to counteract packet losses. Here the symbols the code operates on are not bits, but packets of bits all having the same length. Throughout this document, by symbol we denote either a bit or a packet of bits. We will usually assume a bit-oriented perspective, the extension to packets of bits being straightforward. Therefore, the code is usually referred to as a packet-oriented erasure correcting code (or packet erasure codes). As an example, \(k\) data packets can be encoded

\(^1\) Note that iterative decoding of LDPC codes over the BEC is equivalent to solving \((1)\) through an iterative algorithm processing one equation at time.

\(^2\) The set of positions of a binary vector corresponding to the 1 bits is known as the support of the vector. For an LDPC code, the subset of VNs corresponding to the support of any codeword is a stopping set for the iterative decoder (formal proof available in [16]). These stopping sets cannot be resolved by an ML decoder as they lead to a matrix \(H_{U}\) whose rank is smaller than the number of unknowns of \((1)\). On the other hand, the stopping sets of the iterative decoder which do not include the support of any codeword can be resolved by an ML decoder.)
through a systematic \((n, k)\) packet-oriented code, producing \(n-k\) redundant packets. Each of the \(n\) packets is encoded into a physical layer frame by a bit-oriented error correcting code (e.g., convolutional code, Reed-Solomon code, turbo code, etc.) and by an error detection code (CRC). On the receiver side, after physical layer decoding, the undecodable frames are usually discarded, leading to packet losses at the upper layers. However, if the number of lost packets is low enough, the missing packets can be restored by the erasure decoder. Examples of applications for which the use of packet erasure codes is currently foreseen are listed next.

- **Wireless video/audio streaming.** Link-layer coding is currently applied to the video streams within the framework of the DVB-H/SH standards. In such a context, packet erasure codes take care of fading mitigation, which is crucial especially in the case of mobile users, in challenging propagation environments (urban/suburban and land-mobile-satellite channels). A capacity-approaching performance is highly desirable to increase the service availability. Mobile applications require low-complexity decoders as well.

- **File delivery in broadcasting/multicasting networks.** Reliable file delivery in broadcasting/multicasting networks finds a very favourable solution in erasure correcting codes. In such a scenario, reliability cannot be guaranteed by any automatic repeat request (ARQ) mechanism, due to the broadcast nature of the channel. Packet erasure codes would limit (or avoid) the usage of packet retransmissions.

- **File delivery in point-to-point communications.** Also in point-to-point links, file delivery may require further protection at upper layers. This is true especially if retransmissions are impossible (due to the absence of a return channel or due to long round-trip delays).

- **Deep space communications.** Deep space communication has been always an ideal application field for error correcting codes. The Consultative Committee for Space Data Systems (CCSDS) is currently investigating the adoption of packet erasure codes to further protect the telemetry down-link, especially for deep-space missions, which are not suitable for ARQ. A mandatory feature is instead represented by low-complexity encoder implementations.

- **Power Line Communications.** Packet erasure codes are currently under evaluation for possible inclusion in power line communication standards, as an effective solution to combat the impulse noise present in the power line channel.

### 2.2.3 LDPC codes for packet erasure recovery

#### 2.2.3.1 Efficient ML decoding of LDPC codes over erasure channels

Efficient implementation of Gaussian elimination over sparse matrices of large size and constructed on finite fields is a widely investigated topic (e.g., [25][26]). An effective approach (sometimes referred to as **structured Gaussian elimination**) consists of converting the system of sparse linear equations into a non-sparse system whose unknowns form a small subset of the original set of unknowns. These unknowns are referred to as the **pivots** or **reference variables** and are chosen in such a way that their knowledge is sufficient to resolve all the other unknowns by simple back-substitution operations. Therefore, to solve the linear system it is sufficient to run a brute force Gaussian elimination only to recover from the pivots [24]. It is worth observing that ML decoding over a binary erasure channel can be also performed using a bit guessing approach [27]. This approach, however, is not practical over packet erasure channels, as in this case all the bits composing an erased packet should be guessed.

Within the framework of ML decoding of LDPC codes over erasure channels, the idea of structured Gaussian elimination has been applied in [24]. A brief overview of this approach is reviewed next. For the sake of clarity, let's apply column permutations to arrange the parity check matrix \(H\) as in (1): the left part shall contain all the columns related to known variable nodes \(H_{K}\), whereas the right part shall be made up of all the columns related to erased variable nodes \(H_{U}\). Thus, to solve the unknowns, we proceed as follows:

- **Perform diagonal extension steps on** \(H_{U}\). This results in the sub-matrices \(B\), as well as \(P\) that is in a lower triangular form, and columns that cannot be put in lower triangular form (columns of matrices \(A\) and \(S\)). The variable nodes corresponding to the former set of columns form the above-mentioned pivots (or reference variables) (see Figure 4(a)).

- **Zero the matrix** \(B\) through row summations only. All the remaining unknown variables can be now obtained by linear combination of pivots and known variables only. (Figure 4(b)).
• Resolve the pivots by performing brute-force Gaussian elimination only on the rows of system involving $A'$. If the pivot recovery step has been successful, the remaining unknown variables can be easily obtained due to the lower triangular structure of $P$.

![Figure 4: ML decoding of LDPC codes. (a) Pivots selection within $H_U$. (b) Zeroing of matrix $B$. (c) Gaussian Elimination on $A'$.](image)

The main strength of this algorithm lies in the fact that Gaussian elimination is performed only on $A'$ and not on the entire set of unknown variables. Therefore, it is of great interest to keep the dimensions of $A'$ as small as possible. This can be obtained by both sophisticated ways of choosing the pivots [24] and by a judicious code design [15]. Besides, to reduce the complexity further the brute-force Gaussian elimination step on $A'$ could be replaced by other algorithms.

Note that the ML decoder for an $(n, k)$ LDPC code operates on a sparse matrix with at most $n-k$ columns and $n-k$ rows. The relevance of this consideration will become more clear after the description of the ML Raptor decoder [20] provided later in this document.

### 2.2.3.2 On the code design

A common way to design an LDPC code, with a certain code rate $R$, for the BEC consists of the selection of a proper degree distribution pair [28] (or protograph [29]) with design rate $R$ and offering a satisfying iterative decoding threshold $\epsilon_{IT}$ subject to eventual further constraints (in terms, for instance, of growth rate of the stopping set size distribution [30]). An $(n, nR)$ LDPC code is then picked from the ensemble defined by the above-mentioned degree distribution pair (or protograph). The selection may be performed following some girth optimization techniques. Such a design technique does not necessarily address the need to a good LDPC code for ML decoding. For example, we can look at Table 3, where the asymptotic thresholds over the BEC of some regular LDPC ensembles (with rate 1/2 and 2/3), under both IT decoding ($\epsilon_{IT}$) and ML ($\epsilon_{ML}$) decoding are reported. For both rates, we see that the larger $\epsilon_{IT}$ the smaller $\epsilon_{ML}$. Note that the improvement given by the ML decoder is usually large.

<table>
<thead>
<tr>
<th>Ensemble</th>
<th>$\epsilon_{ML}$</th>
<th>$\epsilon_{IT}$</th>
<th>$\epsilon_{Sh}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>(3,6)</td>
<td>0.4881</td>
<td>0.4294</td>
<td>0.5000</td>
</tr>
<tr>
<td>(4,8)</td>
<td>0.4977</td>
<td>0.3834</td>
<td>0.5000</td>
</tr>
<tr>
<td>(5,10)</td>
<td>0.4994</td>
<td>0.3416</td>
<td>0.5000</td>
</tr>
<tr>
<td>(6,12)</td>
<td>0.4999</td>
<td>0.3075</td>
<td>0.5000</td>
</tr>
<tr>
<td>(3,9)</td>
<td>0.3196</td>
<td>0.2828</td>
<td>0.3333</td>
</tr>
<tr>
<td>(4,12)</td>
<td>0.3302</td>
<td>0.2571</td>
<td>0.3333</td>
</tr>
<tr>
<td>(5,15)</td>
<td>0.3324</td>
<td>0.2303</td>
<td>0.3333</td>
</tr>
</tbody>
</table>

Therefore, within the context of LDPC code design for ML decoding, a different figure shall be put in the focus of the degree distribution optimization, namely, the ML decoding threshold $\epsilon_{ML}$. A method for deriving a tight upper bound on the ML threshold for an LDPC ensemble has been developed in [31]. The upper bound on $\epsilon_{ML}$ can be derived as follows.

• Consider an LDPC code randomly drawn from the ensemble $C(n, \lambda, \rho)$, where $n$ is the codeword length and $(\lambda, \rho)$ is the degree distribution pair [28]. The extrinsic information transfer (EXIT) function [32], [33] under iterative decoding can be derived in terms of extrinsic erasure probability at the output of the decoder (denoted by $p_E$) as a function of the a priori erasure probability input to the decoder (denoted by $p_A$). In the limit where $n \to \infty$, the EXIT function of the ensemble defined by the degree distribution pair $(\lambda, \rho)$ is a function of $(\lambda, \rho)$. It can be expressed in parametric form by the pair of simultaneous equations
\[ p_A = \frac{x}{\lambda(1 - \rho(1 - x))} \]  

(2)

\[ p_E = \Lambda(1 - \rho(1 - x)) \]  

(3)

where \( x \in [x_{BP}, 1] \), \( x_{BP} \) being the value of \( x \) for which \( p_A = \varepsilon_{BP} \), and where \( \Lambda(x) = \sum \Lambda_i x^i \), \( \Lambda_i \) being the fraction of degree-\( i \) VNs. The EXIT functions of two regular LDPC code ensembles are displayed in Figure 5 (dashed lines).

\[ \int_{p_A} p_E(p_A)dp_A = R \]

Figure 5: EXIT functions for the (3,6) and the (5,10) regular LDPC ensembles. Dashed lines represent the iterative decoder EXIT function. Solid lines are placed in correspondence of the ML thresholds upper bounds.

- Due to the area theorem [33, Theorem 1], the area below the EXIT function under ML decoding, must equal the code rate \( R \). As the EXIT function defined by (2) and (3) assumes iterative decoding, the area below the corresponding EXIT curve is larger than the code rate.

- Consider the extrinsic erasure probability at the output of an ML and of an iterative decoder. Obviously, \( p_E^{ML} \leq p_E^{IT} \).

- Therefore, by drawing a vertical line on the EXIT function plot of the ensemble, in correspondence with \( p_A = p_A^* \) and such that

\[ \int_{p_A} p_E(p_A)dp_A = R \]

we obtain an upper bound on the ML threshold, i.e., \( \varepsilon_{ML} \leq p_A^* \) (see an example in Figure 5 for two regular ensembles).

It was illustrated in [31] hat this bound on \( \varepsilon_{ML} \) is very tight for regular LDPC ensembles, and for ensembles whose EXIT curve under IT decoding presents one jump (for further details, see [31]). It was also illustrated how slightly different (but still rather simple) techniques to obtain tight bounds are applicable also in the other cases. Extensions of the above-mentioned techniques can be applied to other code ensembles, provided the EXIT curve under iterative decoding is available. For example, For protograph LDPC ensembles, a rather simple approach consists of applying
EXIT analysis for protograph to obtain the EXIT curve under iterative decoding for a given protograph ensemble [34]. The upper bound on the ML threshold can then be obtained as for the standard LDPC ensemble defined by a degree distribution pair. An example of the EXIT curve under iterative decoding for an accumulate-repeat-accumulate (ARA) ensemble [35] is depicted in 15 (specifically, for the ARA ensemble with repetition rate 3, referred to as AR3A ensemble), together with the derivation of the corresponding upper bound on $\varepsilon_{\text{ML}}$.

Figure 6: EXIT function for the accumulate-repeat-accumulate (ARA) ensemble. Dashed lines represent the iterative decoder EXIT function. Solid lines are placed in correspondence of the ML thresholds upper bounds.

For the regular ensembles, the improvement given by the ML decoder is usually large (see Table 3). A rule of thumb for the design of capacity-approaching LDPC codes under ML consists in the selection of sufficiently dense parity-check matrices, by keeping for instance a relatively large average check node degree. To given an idea, for rate 1/2 LDPC ensembles, an average check node degree $d_c \geq 9$ is sufficient to provide ML thresholds close to the Shannon limit [15]. This heuristic rule seems to work for both regular and irregular ensembles.

Overall, the design of LDPC codes for ML decoding requires consideration of severe constraints and complexity issues. The aim is to generate codes with a manageable encoding/decoding complexity and exhibiting a near-optimum performance down to low error rates. The following general requirements should be fulfilled.

- **R1.** The code shall be systematic.
- **R2.** Low-complexity encoding shall be guaranteed.
- **R3.** The frequency of failure of the diagonal extension step performed on $H_U$ shall be as small as possible even for large erasure rates due to the channel.
- **R4.** When the diagonal extension step performed on $H_U$ fails, the number of reference bits shall be as small as possible.
- **R5.** The performance under ML decoding shall closely match that of an idealized MDS code in the waterfall region.
- **R6.** The code shall exhibit a low error floor.

Systematic codes allow delivery of the correctly received information bits even in case of decoding failure (R1). Efficient encoding is required to obtain a coding scheme easy to implement (R2). The code shall be designed in order to

3 For given $k$ and $n$ we use as a benchmark the performance under ML decoding of an idealized MDS code with minimum distance $d_{\text{min}}=n-k+1$. Note that, in general, this code does not exist.
minimize the decoding complexity (R3 and R4). From a code design viewpoint, the requirements R3 and R4 imply that the LDPC code shall exhibit a good iterative decoding threshold \( \varepsilon_{IT} \) as a larger \( \varepsilon_{IT} \) is usually associated with a smaller number of reference bits. We observe that, while under iterative decoding \( \varepsilon_{IT} \) is related to the waterfall performance of the LDPC code, when using low-complexity ML decoding a larger \( \varepsilon_{IT} \) implies a reduced decoding complexity and vice-versa. In order to have a close-to-ideal waterfall performance (R5) we need to design LDPC codes with ML threshold \( \varepsilon_{ML} \) close to \( 1-R \), which usually requires large check node degrees. Since \( d_{\text{min}} \leq n-k+1 \), the performance deviates from that of an ideal MDS code due to the error floor only depending on the code distance spectrum. Such an error floor appears at low error rates (R6) if the designed code has a good (large) minimum distance.

Note that some of these requirements are conflicting (e.g. R3/R4 and R6), imposing a trade-off. For short \( n \) (e.g., a few hundreds of bits) the requirements R3 and R4 (decoding complexity, related to \( \varepsilon_{IT} \)) can be relaxed. In fact, due to the short codeword length, a more frequent failure of the diagonal extension step and a larger fraction of reference bits can be afforded. On the other hand, the requirement R6 (minimum distance) may become an issue. Near-regular LDPC codes should be used in this regime, where \( \varepsilon_{ML} \) for the near regular distribution shall be very close to \( 1-R \). This requires a larger check node degree than is usually done for iterative decoding, e.g., degree 8 instead of 6 for \( R=1/2 \). For longer codes, the requirements on the decoding complexity (R3/R4) shall be favoured, which imposes consideration of irregular LDPC codes with a larger \( \varepsilon_{IT} \). Again, \( \varepsilon_{ML} \) shall be very close to \( 1-R \).

It is also worthwhile pointing out how, in practical applications, the packet erasure channel is usually not a memoryless erasure channel. In fact, erasures are typically correlated and bursts of erasures often take place. The LDPC code construction technique shall take into account the bursty nature of the packet erasure channel. This can be achieved by proper construction techniques [36], [37], [38], that will be employed within the OPTIMIX Projects.

### 2.2.3.3 GeIRA Approach

Concerning R1 and R2, there are several solutions for a systematic and efficient LDPC encoding. Among them, a very simple one is represented by systematic IRA (SIRA) encoding [39], [40]. The major drawback of this coding technique is poor minimum distance (see [41, Theorem 23]). In order to preserve the extremely simple SIRA-like encoding while improving the minimum distance, GeIRA codes can be considered [42] (see also [45]). They fulfil R1 and R2 while, as shown in the next section, offering a good compromise between R3, R4, R5 and R6. GeIRA codes are systematic LDPC codes that generate the parity bits by a serial concatenation of an outer low-density generator matrix (LDGM) code with an inner rate-1 recursive convolutional code (RCC). Decomposing the parity-check matrix as \( H = [H_s \mid H_p] \), where \( H_s \) corresponds to the \( k \) systematic bits and \( H_p \) to the \( m=n-k \) parity bits, we have that \( H_p^T \) is the outer LDGM code generator matrix. Moreover, \( H_p \) is specified by the feedback polynomial

\[
G(D) = \sum_{j=0}^{t} g_j D^j
\]

of the inner rate-1 RCC (where \( g_j \in \{0,1\} \) and \( g_0 = g_{t} = 1 \)). Correspondingly, \( H_p \) is lower triangular and its ‘1’s have a multi-diagonal structure, where the number of all-‘1’ diagonals equals the number of non-null coefficients of \( g(D) \) (see example in Figure 7).

Note that a SIRA code can be seen as a GeIRA code with \( g(D) = 1+D \). While for SIRA codes the number of degree-2 variable nodes is constrained to be not smaller than the number \( m \) of check nodes, this is not required for GeIRA codes. Allowing multiple diadons in \( H_p \) enables to still employ a highly efficient SIRA-like encoding but with a smaller number of degree-2 variable nodes, and also to gain in flexibility for the choice of the variable node degrees. The reduced number of degree-2 variable nodes is beneficial in terms of \( d_{\text{min}} \) as it is possible to generate both irregular codes with controlled \( d_{\text{min}} \) and near-regular codes exhibiting good \( d_{\text{min}} \) even for short block lengths. Given \( g(D) \), the check node distribution and the systematic variable node distribution, the GeIRA code can be constructed with the following algorithm. The connections for the parity variable nodes are first drawn according to the multi-diagonal structure of \( H_p \). The bipartite graph is then completed with the PEG algorithm [43] for the systematic variable nodes.
2.2.4 MBMS Raptor codes for packet erasure recovery

As mentioned previously, Raptor codes were introduced in [17]. They are an instance of the concept of fountain code\(^4\) [19] and, thanks to the large degrees of freedom in parameter choice; they can be applied to several systems, increasing their reliability. Recently, a fully specified version of Raptor codes has been approved to efficiently disseminate data over a broadcast network (MBMS service [20, Annex B]). Fixed-rate Raptor codes derived from the MBMS standard are currently under investigation for the multi protocol encapsulation (MPE) level protection within the DVB standards family [21]. In the following, a description of the Raptor codes specified in [20, Annex B] is provided, including some insights on their encoding and recommended decoding algorithms. Nevertheless it must be kept in mind that other realisation of Raptor Codes (or even Fountain Codes) will likely also be considered in OPTIMIX project, if only in order to further estimate the interest of this standard specified one.

The Raptor code can be viewed as the concatenation of several codes. Let’s consider the systematic Raptor encoder specified in [20] that is also depicted in Figure 8. The most-inner code is a non systematic Luby-transform (LT) code [22] with \(L\) input symbols \(F\), producing the encoded symbols \(E\). The symbols \(F = [D_T | D_s^T | D_h^T]^T\) are known as the intermediate symbols, and are generated through a pre-coding, made up of some outer high-rate block coding, effected on the \(k\) symbols \(D\) The \(s\) intermediate symbols \(D_s\) are known as the LDPC symbols, while the \(h\) intermediate symbols \(D_h\) are known as the half symbols. The combination of pre-code and LT code produces a non systematic Raptor code. The parameters \(s\) and \(h\) are functions of \(k\), according to [20]. Some pre-processing is to be put before the non-systematic Raptor encoding to obtain a systematic one. Such a pre-processing consists of a rate-1 linear code generating the \(k\) symbols \(D\) from the \(k\) information symbols \(C\).

LT codes are the first practical implementation of fountain codes. A unique encoded symbol ID (ESI) is assigned to each encoded symbol. Starting from an ESI \(i\), the encoded symbol \(E_i\) is computed by XOR-ing a subset \(\Theta_i\) of \(d_i\) intermediate symbols. The number \(d_i\), known as the degree associated with the encoded symbol \(E_i\) and is a random integer between 1 and \(L\): the \(d_i\) intermediate symbols are chosen at random according to a specific probability distribution. As a consequence, in order to recover the information symbols the decoder needs both the set of encoded symbols \(E_i\) and of the corresponding \(\Theta_i\). This last information can either be explicitly transmitted or obtained by the decoder through the same pseudo-random generator used for the encoding, starting from ESIs, which have therefore to be sent together with the corresponding encoded symbols (as depicted in Figure 8).

Some of the main properties of LT codes are that the encoder can generate as many encoded symbols as desired and that the decoder is able to recover the block of source symbols from any set of received encoded symbols whose number is only slightly greater than that of the source symbols (in fact the code claims a low amount of overhead). A Raptor code, whose core consists of an LT code, inherit such properties.

\(^4\) Commonly, the expression “fountain code” is used to refer to a code which can produce on-the-fly any desired number of encoded symbols from \(k\) information symbols (see also [18]).

\(^5\) An \((n, k)\) fixed-rate Raptor code is obtained by limiting to \(n\) the amount of symbols produced by the Raptor encoder.
2.2.4.1 Generator matrix

Considering a systematic Raptor code as a finite length \((n, k)\) linear block code (fixed-rate Raptor code \(^6\)), we can ask what is the structure of its generator matrix. This problem is addressed next for the Raptor code specified in [20] \(^7\). The generator matrix of the first pre-coding stage is given by \([I_k \mid G_{LDPC}^T]^T\). According to the specifications in [20], \(G_{LDPC}\) consists of columns all of weight equal to 3, regardless the value of \(k\). On the other hand, the generator matrix of the second pre-coding stage is given by \([I_{s+k} \mid G_H^T]^T\), where \(G_H\) is a \((h \times (s+k))\) matrix consisting of columns all of constant weight: each column is an element of the Grey sequence of weight \(h'\), where \(h'=\lfloor h/2 \rfloor\). Finally, let us denote by \(G_{LT}\) the \((n \times L)\) LT code generator matrix (regarded as a finite length \(n\) linear block code). It is built in such a way that the row of index \(i\) has \(d_i\) ones in \(\Theta_i\) positions, where \(d_i\) and \(\Theta_i\) are derived from the ESI \(i\), through pseudo-random algorithms described in [20]. Next, we use the notation \(G_{LT}(i_1, i_2, \ldots, i_r)\) to denote the \((r \times L)\) submatrix of \(G_{LT}\) composed of the rows with indexes \((i_1, i_2, \ldots, i_r)\). The notation \(G_{LT}\) is equivalent to \(G_{LT}(1, \ldots, n)\).

The \(L=k+s+h\) intermediate symbols \(F\) are obtained from \(D\) as

\[
F = \begin{bmatrix} D \\ D_s \\ D_h \end{bmatrix}
\]

through the relations

\[
D_s = G_{LDPC} \cdot D \tag{4}
\]

\[
D_h = G_H \cdot \begin{bmatrix} D \\ D_s \end{bmatrix} \tag{5}
\]

The intermediate symbols \(F\) are the inputs to the LT encoder for deriving the \(n\) encoded symbols \(E\) as

\[
E = G_{LT} \cdot F \tag{6}
\]

Let us subdivide \(G_{LT}\) as

\(^6\) For rate-less applications just think of \(n\) as the maximum possible number of transmitted symbols.

\(^7\) Throughout this section, the vectors are intended as column vectors (unless explicitly mentioned) and the generator matrix of a \((n, k)\) linear block code is expressed as a \((n \times k)\) matrix.
\[ G_{LT} = \begin{bmatrix} G_{LT}^I | G_{LT}^II | G_{LT}^III \end{bmatrix} \]

where the sizes of the three submatrices are \((n \times k)\), \((n \times s)\) and \((n \times h)\), respectively. If also \(G_H\) is subdivided as

\[ G_H = \begin{bmatrix} G_H^I | G_H^II \end{bmatrix} \]

that is into two submatrices whose sizes are \((h \times k)\) and \((h \times s)\), respectively, then the non-systematic Raptor code generator matrix can be expressed as

\[ G_{R,n-sys} = G_{LT}^I + G_{LT}^II \cdot G_{LDPC} + G_{LT}^III \cdot (G_H^I + G_H^II \cdot G_{LDPC}) \]

which satisfies the relation:

\[ E = G_{R,n-sys} \cdot D \]

Let’s now subdivide \(G_{R,n-sys}\) into the two submatrices \(G_{R,n-sys}^I\) and \(G_{R,n-sys}^II\), whose sizes are \((k \times k)\) and \(((n-k) \times k)\), respectively:

\[ G_{R,n-sys} = \begin{bmatrix} G_{R,n-sys}^I \\ G_{R,n-sys}^II \end{bmatrix} \]

For a systematic code it must be valid the following

\[ E_j = C_j \quad \forall i = 1, \ldots, k \]

and therefore

\[ \begin{bmatrix} G_{R,n-sys}^I \\ G_{R,n-sys}^II \end{bmatrix} \cdot D = \begin{bmatrix} E_{[1 \ldots k]} \\ E_{[k+1 \ldots n]} \end{bmatrix} = \begin{bmatrix} C \\ E_{[k+1 \ldots n]} \end{bmatrix} \]

(7)

We have introduced in (7) the notations \(E_{[1 \ldots k]}\) and \(E_{[k+1 \ldots n]}\) to denote the first \(k\) and the last \(n-k\) encoded symbols, respectively. We can state that the pre-processing matrix generating \(D\) from \(C\) can be obtained by

\[ G_T^{-1} = (G_{R,n-sys}^I)^{-1} \]

and, as a consequence, the systematic Raptor code generator matrix is

\[ G_{R,sys} = \begin{bmatrix} I_k \\ G_{R,n-sys}^II \end{bmatrix} \]

(8)

In (8) \(I_k\) denotes the \((k \times k)\) identity matrix. Obviously, \(G_{R,n-sys}^I\) can be inverted if and only if it has full rank \(k\). By initializing the random generator of inner LT code through the so-called systematic index (defined in [20]), this property is guaranteed to be fulfilled for all \(k=4, \ldots, 8192\).

2.2.4.2 Encoding

The relations (4), (5) and (6) can conveniently be represented as:

\[ A \cdot F = \begin{bmatrix} 0 \\ E_{[1 \ldots n]} \end{bmatrix} \]

where \(A\) is a \(((s+h+n) \times (s+h+k))\) binary matrix called the encoding matrix, whose structure is shown in Figure 9. In such a figure, \(I_s\) is the \((s \times s)\) identity matrix, \(I_h\) is the \((h \times h)\) identity matrix and \(Z\) is the \((s \times h)\) all-zero matrix. The matrix \(A\) doesn’t properly represent the Raptor code generator matrix (which is defined in (7) instead), but includes the set of constraints imposed by the pre-coding and LT coding together. We use next the notation \(A(i_1, i_2, \ldots, i_s)\) to indicate
the \(((s+h+r) \times L)\) submatrix of \(A\) obtained by selecting only the rows of \(G_{LT}\) with indexes \((i_1, i_2, \ldots, i_r)\). Again, \(A\) is equivalent to \(A(i_1, \ldots, n)\).

A possible Raptor encoding algorithm exploits a submatrix of \(A\). Such a matrix, consisting of the first \(L\) rows of \(A\), is used to obtain \(F\) solving the system of linear equations:

\[
A(i_1, \ldots, k) \cdot F = \begin{bmatrix} \theta \\ C \end{bmatrix}.
\]

At this point it is sufficient to multiply \(F\) by the LT generator matrix to produce the encoded symbols \(E\), according to (6).

\[
\begin{bmatrix} k \\ s \\ h \end{bmatrix} \begin{bmatrix} G_{LDPC} & I_s & Z \\ G_H & I_h & \end{bmatrix} \begin{bmatrix} s \\ s \\ h \end{bmatrix} = \begin{bmatrix} n \end{bmatrix}
\]

\begin{figure}[h]
\centering
\includegraphics[width=0.5\textwidth]{encoding_matrix.png}
\caption{Structure of the encoding matrix \(A\) for an \((n, k)\) Raptor code specified in [20] \((L=k+s+h)\).}
\end{figure}

### 2.2.4.3 Recommended Decoding

The most direct way to decode the received sequence lies in inverting each encoding step of Figure 8; in this case you work on individual sub-codes. When using ML decoding at each sub-code, such a method requires the inversion of a matrix for each code, so it doesn't appear to be the best solution from the computational viewpoint [23]. Moreover, if the number of received encoded symbols is not larger enough than (which in many cases may mean much higher than) the number of source symbol \(k\), it shows an high failure probability.

For example, let's assume that only a subset of encoded symbols of ESIs \((i_1, i_2, \ldots, i_r)\) are available at the decoder. The first step the decoder should perform is to solve the system of linear equations:

\[
G_{LT}(i_1, \ldots, i_r) \cdot F = E_{\{i_1, \ldots, i_r\}}
\]

The matrix \(G_{LT}(i_1, i_2, \ldots, i_r)\) has \((r \times L)\) size and, obviously, the necessary condition to solve the system is that \(r \geq L\). If such a condition is not fulfilled, the decoding fails. It means that to recover the source symbols the decoder requires at least \(L\) encoded symbols (let's recall that \(L=k+s+h\)).

Such a method doesn't exploit the fact that the \(L\) intermediate symbols are not independent from each other, but subject to the pre-coding constraints, instead. Therefore, to obtain the intermediate symbols \(F\) by using a submatrix of \(A\) (which consider such constraints) turns out to be a far more efficient solution.

According to the above-mentioned assumption, the first decoding step will turn into

\[
A(i_1, \ldots, i_r) \cdot F = \begin{bmatrix} \theta \\ E_{\{i_1, \ldots, i_r\}} \end{bmatrix}
\]

where \(A(i_1, i_2, \ldots, i_r)\) is a \(((s+h+r) \times L)\) matrix, as defined above. The system can be solved by Gaussian elimination (ML decoding) only if \(s+h+r \geq L\), that is \(r \geq k\) (note that this is a necessary condition for successful ML decoding, not a sufficient one). In this way the number of encoded symbols required at the decoder is definitely lower compared to that
in the previous case and, notably, is close to the number of source symbols \( k \). Once \( F \) is known, the source symbols \( F \) are easily recovered by
\[
C = G_{LT}(1, ..., k) \cdot F
\]

To sum up, when the described encoding and decoding algorithms are employed, both the encoding and the decoding are performed by making use of operations which are analogous in the two case (Figure 10). The technique to resolve (9) recommended in [20] is described next, where we define
\[
b = \begin{bmatrix} 0 \\ E_{i_1, ..., i_r} \end{bmatrix}.
\]

Note that this procedure can be used also for efficient ML decoding of LDPC codes over the erasure channel (just replace \( A(i_1, i_2, ..., i_r) \) with \( H_U \) and \( b = x_K H_K \)).

![Diagram](image-url)

**Figure 10: Overview of the encoding and decoding process for the systematic Raptor code specified in [20].**

The algorithm modifies the matrix \( A(i_1, i_2, ..., i_r) \) in such a way that, at the generic algorithm step, \( A(i_1, i_2, ..., i_r) \) is partitioned as shown in Figure 11. At the beginning of the algorithm the matrix \( V \) coincides with \( A(i_1, i_2, ..., i_r) \). At each step of the algorithm, one row of \( V \) is selected. Specifically, one minimum weight row of \( V \) is selected. Two possible cases are distinguished:

A. the minimum row weight of \( V \) is 1 or larger than 2;

B. the minimum row weight of \( V \) is 2.

![Diagram](image-url)

**Figure 11: Modification of \( A(i_1, i_2, ..., i_r) \) performed by the reduced complexity ML decoder recommended in [20].**

Let us consider the case A first. If several minimum weight rows of \( V \) are present, then the selected row is that having minimum weight in the \( A(i_1, i_2, ..., i_r) \) matrix. If several such rows are present, one of them is selected randomly.
The algorithm behaviour in the case B is a little more elaborated. Regardless of its weight, the first row of \( V \), here denoted by \( r_1 \), is copied into the first row of a matrix \( V_1 \) having the same number \( v \) of columns as \( V \). A binary vector of length \( v \), namely \( f_j \), is also set equal to \( r_1 \). Among all the other rows of \( V \), those having at least a `1’ position overlapping with a `1’ position of \( r_1 \) are identified, and a copy of each of these rows is stacked as a row of \( V_1 \) together with \( r_1 \). All the \( V \) rows copied into \( V_1 \) are identified by a specific flag. All the zero elements of \( j \) which correspond to a non-zero column of \( V_1 \) are set to 1. Once \( f_j \) has been updated, the rows of \( V \) not yet stacked into \( V_1 \) and having at least a `1’ position overlapping with a `1’ position of \( f_j \) are searched, and a copy of each of them is stacked as a row of \( V_1 \).

The described procedure is iterated until either \( V_1 = V \) or \( V_1 \neq V \) but the algorithm is no longer able to stack new rows of \( V \) into \( V_1 \). In this latter case the whole procedure is repeated by copying the first of the remaining rows of \( V \), namely \( r_2 \), into the first row of a matrix \( V_2 \), by setting a vector \( f_j \) to \( r_2 \) and then by building \( V_2 \) iteratively.

At the end of the whole procedure, all the rows of \( V \) have been partitioned into several matrices \( V_i \). Among all the weight-2 rows of \( V_i \) that one belonging to the matrix \( V_i \) having the largest number of rows is selected. If there are several such weight-2 rows, or if there are several matrices \( V_i \) with the largest number of rows and containing weight-2 rows, one of such weight-2 rows is selected randomly.

In either the case A, or the case B, one specific row of \( V \), having weight \( w \), is selected. The following operations are then performed. First, the \( A(i_1, i_2, \ldots, i_r) \) rows are permuted in order to let the selected row be the first row of \( V \) (the same permutation is applied to \( b \)). Second, the \( A(i_1, i_2, \ldots, i_r) \) columns are permuted in order to have a `1’ of the selected row at the top-left position of \( V \) and the other \( w-1 \) `1’s of this row at the top-right positions of \( V \). Third, all the `1’s below the `1’ at the top-left of \( V \) are made equal to `0’ by row summations only (the same row summations are applied to \( b \)). Finally, the left-most column of \( V \) as well as its top row are no longer considered as part of \( V \) (\( i \) is increased by 1 and \( v \) is decreased by 1); furthermore, the \( w-1 \) columns on the left of \( U \) are included into \( U \) (\( u \) is increased by \( w-1 \) and \( v \) is decreased by \( w-1 \)).

![Figure 12: Structure of \( A(i_1, i_2, \ldots, i_v) \) at the end of the modification procedure.](image)

The algorithm terminates when \( v=0 \). At the end of this modification procedure, the matrix \( A(i_1, i_2, \ldots, i_v) \) assumes the aspect depicted in Figure 12, where the last \( p \) equations involve only the unknowns corresponding to the \( u \) columns of \( U \) (reference symbols). These unknowns are recovered by applying Gaussian elimination to the submatrix composed of the last \( p \) rows and the last \( u \) columns of \( A(i_1, i_2, \ldots, i_v) \). This operation can be performed with low complexity as far as the number of reference symbols is sufficiently small. Indeed, the described modification procedure is designed in order to keep \( p \) as small as possible, in order to reduce the complexity of the Gaussian elimination step, which dominates the overall decoding complexity. Once the reference symbols have been recovered, each of the other symbols can be obtained from the first \( i \) equations.

### 2.2.4.4 Some remarks on the decoding complexity of LDPC and fixed-rate Raptor codes

The above described algorithm to perform efficient Gaussian elimination on \( A \) shares some similarities with the one proposed in [24] for LDPC codes. In both cases, the erased symbols are solved by mean of a structured Gaussian elimination, exploiting the sparse nature of the equations to reduce the size of the matrix on which brute-force Gaussian elimination is performed. The targets of the structured Gaussian elimination are \( H_U \) for LDPC codes and \( A \) for Raptor codes. Consider now an \((n,k)\) LDPC code and its (fixed-rate) Raptor counterpart. Suppose also an erasure pattern (introduced by the communication channel) leading to a small overhead \( \delta \), i.e., that the amount of correctly received symbols is \( k+\delta \). On the LDPC code side, the structured Gaussian elimination will be performed on \( H_U \) with size \((n-k) \times \)
(n-k-\delta)). For the Raptor code, the structured Gaussian elimination will work on A with size ((k+ \delta+s+h) \times (k+s+h)). Hence, while for the LDPC code the complexity of the ML decoder is driven by (n-k) (i.e., the amount of redundancy, thus by the code rate \( R \)), for the Raptor code the complexity depends just on \( k \) (i.e., it’s code rate independent). The result is that for high rates (\( R>1/2 \)) LDPC codes have an inherent advantage in complexity. On the other hand, for lower rates Raptor codes shall be preferable from a complexity viewpoint.

It must also be pointed out that usage of Belief Propagation, albeit less efficient than Gaussian elimination, can also be considered for Raptor codes. Larger Raptor codes may in practice need to consider Belief Propagation if decoding complexity is an issue.

### 2.2.5 Preliminary Numerical Results

The performance of some moderate-length LDPC codes is provided in Figure 13, Figure 14 and Figure 15. In Figure 13, the CER for a (2048, 1024) GeIRA code from [15] is presented. The code is picked from an LDPC ensemble with \( \varepsilon_{ML}=0.480 \) and \( \varepsilon_{ML}=0.496 \). The code performance, under ML decoding, tightly approaches the Singleton bound, and practically matches the Berlekamp bound. The iterative decoding curve, although not so far from the state-of-the-art for iteratively-encoded codes, lies quite far from the bound. The sub-optimality of the IT curve is therefore not due to the code by itself, but to the sub-optimality of the decoder. The result is confirmed for a family of rate-compatible GeIRA codes with code rates ranging from 1/2 to 4/5 and input block size \( k = 502 \) (Figure 14). The higher rates are obtained by puncturing the mother \( R=1/2 \) code, which has been derived from the construction proposed in [15]. For the code rates under investigation, the performance is uniformly close to the corresponding Singleton bound, down to low codeword error rates (\( \text{CER} \approx 10^{-6} \)). We remark that rate-compatibility allows using the LDPC codes as fountain codes, with the only limitation due to the lowest possible code rate which is given by the mother code. In Figure 15, the codeword error rate for a (1160, 1044) \( R=9/10 \) is shown. The code is a near-regular GeIRA code with almost constant column weight \( w_c=5 \) and feedback polynomial given by \( g(D)=1+D^4+D^{10}+D^{50} \). The ML threshold is \( \varepsilon_{ML}=0.0994 \), while \( \varepsilon_{ML}=0.0699 \). Also in this case, the error performance curve matches the Berlekamp bound down to low error rates. The minimum distance of this code (and its corresponding multiplicity) has been evaluated by [44]. An error floor estimation has been carried out by mean of the truncated union bound on the codeword error probability. Four codewords at \( d_{min}=11 \) have been found, leading to the error floor estimation provided in Figure 15. Even if such results represent only an estimation of the actual error floor, they are quite remarkable. The code performance would in fact deviate remarkably from the Singleton bound just at error rates below \( 10^{-14} \).

A comparison between LDPC codes and Raptor codes specified in the MBMS standard is also provided. In Figure 16, the decoding failure probability (i.e., the CER) as a function of the overhead is depicted for the codes specified in [20] and for some GeIRA codes. The overhead \( \delta \) here defined as the number of encoded symbols that are correctly received in excess with respect to \( k \) (recall that \( k \) represents the minimum amount of correctly-received encoded symbols allowing successful decoding with an ideal MDS code). We see that there is basically no difference in performance between the MBMS Raptor codes and properly-designed LDPC codes under ML decoding. As already pointed out in [46], the decoding failure probability vs. overhead does not seem to depend on the input block size. A comparison between a (512,256) fixed-rate Raptor code and a near-regular GeIRA code from [15] with constant column weight \( w_c=4 \) is provided in Figure 17. In the waterfall region the two codes exhibit almost the same performance. A minimum distance estimation according to [44] was conducted on the two codes. For the (512,256) fixed-rate Raptor code, the minimum distance is given by \( d_{min}=25 \), with multiplicity \( A_{min}=2 \). For the GeIRA code, the estimated minimum distance is \( d_{min}=40 \), with multiplicity \( A_{min}=2 \). In both cases, the estimated minimum distance is quite large, and would permit to achieve very low error floors. For the Raptor code, the error floor estimation predicts a deviation from the Berlekamp bound at \( \text{CER} \approx 10^{-11} \), while for the GeIRA code the error floor would appear at \( \text{CER} \approx 10^{-20} \). The later result is quite impressive, and would suggest the use of the near-regular GeIRA construction for applications requiring very low error floors. Let us make a final remark on the minimum distance evaluation for fixed-rate Raptor codes. The minimum distance evaluation has been applied to fixed-rate MBMS Raptor codes with various block lengths. For a (128, 64) Raptor code, the lowest-weight codeword found by the algorithm described in [44] was \( d_{min}=14 \) (with \( A_{min}=2 \)). In the case of a (2048, 1024) Raptor code, \( d_{min}=26 \) (\( A_{min}=2 \)). Recalling the result for the (512, 256) Raptor code (\( d_{min}=25 \)), it appears from this preliminary analysis that for fixed-rate Raptor codes the minimum distance might scale sub-linearly with the block length.
Figure 13: Codeword error rate for a (2048,1024) GeIRA code. The solid line represents the Singleton bound on the CER.

Figure 14: Codeword error rates for a family of GeIRA codes with input block size $k=502$ and code rates spanning from $1/2$ to $4/5$. The solid lines represent the respective Singleton bounds on the CER, while dotted lines represent the respective Berlekamp random coding bounds.
Figure 15: Codeword error rate for a (1160,1044) GeIRA code. The performance is compared to the Berlekamp bound and to the Singleton bound.

Figure 16: Codeword error rate vs. overhead $\delta$ for the MBMS Raptor code and for some GeIRA codes, various input block size.
2.3 (Rate compatible) LDPC codes for physical layer FEC: possibility of UEP through LDPC

Another of the forward error correction (FEC) schemes that we will employ in the OPTIMIX chain is based on a LDPC codec. A rate-compatible coding scheme [91],[92] has been implemented and the corresponding co-decoder will be included in the common OMNeT++ simulation chain. The considered LDPC permits the choice of the final code rate within 5 options: 1/3 (denoted as scheme 0), 1/2 (scheme 1), 2/3 (scheme 2), 3/4 (scheme 3) and 5/6 (scheme 4).

The code family keeps the codeword length constant (i.e., $n$ is constant, while the code rate adaptation is obtained by a proper choice of the information block length, $k$). In our implementation, the code has been designed choosing $n=4200$.

The encoding procedure realises both puncturing and shortening of a (10500, 3500) mothercode to achieve the desired code rates and block lengths. The belief propagation (BP) algorithm is based on the bipartite graph of the mother code, with a proper initialization of the channel inputs for the punctured and shortened variable nodes: punctured variable nodes are initialized with an a-priori probability log-likelihood ratio (APP-LLR) equal to 0, while shortened variable nodes are initialized with a large positive APP-LLR (which approximates an infinite-reliable knowledge of the codeword bit associated to the shortened variable node). The iterative decoding of a punctured LDPC code can succeed if the code is properly designed and if the choice of the puncturing patterns does not lead to stopping sets. The mother code is a systematic IRA LDPC code.

<table>
<thead>
<tr>
<th>Scheme</th>
<th>Code rate</th>
<th>Periodic puncturing pattern (parity part)</th>
<th>Shortening range (systematic part)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>1/3</td>
<td>101101010101010101010101</td>
<td>1000-3499</td>
</tr>
<tr>
<td>1</td>
<td>1/2</td>
<td>101101010101101010110101</td>
<td>2000-3499</td>
</tr>
<tr>
<td>2</td>
<td>2/3</td>
<td>011111110111111111111111</td>
<td>2900-3499</td>
</tr>
<tr>
<td>3</td>
<td>3/4</td>
<td>111111111111111111111111</td>
<td>3150-3499</td>
</tr>
<tr>
<td>4</td>
<td>5/6</td>
<td>011111111101101111111111</td>
<td>None</td>
</tr>
</tbody>
</table>

In general, there is an important difference between a scheme performing UEP through RCPCCs and one based on LDPC codes: with a solution based on convolutional codes, the transmitted codewords can be variably long and their structure can reflect the data partitioning of protection levels for the corresponding data. Clearly the decoder requires the knowledge of the de-puncturing matrices to apply and the positions where to change them along the received sequence. On the contrary, with the proposed UEP scheme based on LDCCCs, the protection inside each packet is constant and determined by the selected coding scheme. Moreover, the packets to encode should have a predetermined length, according to the values of $k$ in Error ! Source du renvoi introuvable. The latter problem can be easily overcome through shortening, while, in order to manage packets containing data belonging to different priority layers, it would be necessary to rearrange the whole packets structure. For this reason, within the OPTIMIX framework we decided to avoid the possibility to perform UEP at the physical layer by differently protecting distinct data partitions.
within the same packet. Clearly, distinct packets, containing different priority data, can be transmitted exploiting UEP through the described LDPC code.
3 Modulation and Transmission

In a digital communication chain, the channel encoding is traditionally followed by the modulation, which shapes the signal for transmission on the physical medium. Beside very simple and well known modulation, we focus in this chapter on advanced modulations and solutions that come on the border of modulation world such as space-time block coding with sphere packing modulation, optimisation of constellation labelling of bit-interleaved coded modulation with iterative decoding, Multicell cooperation and cooperative relaying.

3.1 Space-Time Block Code with Sphere Packing Modulation (STBC-SP)

3.1.1 General Concept

The concept of combining orthogonal transmit diversity designs with the principle of sphere packing was introduced by Su et al. in [48]. Orthogonal transmit diversity designs can be described recursively [49] as follows. Let $G_i(x_i) = x_iI_2$, and

$$G_{2^k}(x_1, x_2, \ldots, x_{k+1}) = \begin{pmatrix} G_{2^{k-1}}(x_1, \ldots, x_{k+1}) & x_{k+1}I_{2^k-1} \\ -x^*_i I_{2^k-1} & G^H_{2^{k-1}}(x_1, \ldots, x_{k+1}) \end{pmatrix}.$$  

For $k = 1, 2, 3, \ldots$, where $x^*_i$ is the complex conjugate of $x_i$, $G^H_{2^{k-1}}(x_1, \ldots, x_{k+1})$ is the Hermitian of $G_{2^{k-1}}(x_1, \ldots, x_{k+1})$ and $I_{2^k-1}$ is a $(2^{k-1} \times 2^{k-1})$ identity matrix. Then, $G_{2^k}(x_1, x_2, \ldots, x_{k+1})$ constitutes an orthogonal design of size $(2^{k-1} \times 2^{k-1})$, which maps the complex variables representing $(x_1, x_2, \ldots, x_{k+1})$ to $2^k$ transmit antennas. In other words, $x_1, x_2, \ldots, x_{k+1}$ represent $k+1$ complex modulated symbols to be transmitted from $2^k$ transmit antennas in $T = 2^k$ time slots. It was shown in [48] that the diversity product quantifying coding advantage of an orthogonal transmit diversity scheme is determined by the minimum Euclidean distance of the vectors $(x_1, x_2, \ldots, x_{k+1})$. Therefore, in order to maximize the achievable coding advantage, it was proposed in [48] to use sphere packing schemes that have the best known minimum Euclidean distance (MED) in the $2(k+1)$-dimensional real-valued Euclidean space $\mathbb{R}^{2(k+1)}$ [50]. The results of [48] demonstrated that the proposed Sphere Packing (SP) aided Space-Time Block Coded (STBC) system, referred to here as STBC-SP, was capable of outperforming the conventional orthogonal design based STBC schemes of [51][52].

3.1.2 $G_2$ Orthogonal Design Using Sphere Packing

This section describes the STBC-SP scheme proposed in [48], considering space-time systems employing two transmit antennas, where the space-time signal is given by [51]

$$G_2(x_1, x_2) = \begin{pmatrix} x_1 & x_2 \\ -x^*_2 & x^*_1 \end{pmatrix},$$  

(10)

and the rows and columns of Equation (10) represent the temporal and spatial dimensions, corresponding to two consecutive time slots and two transmit antennas, respectively. According to Alamouti’s design [51] for example, $x_1$ and $x_2$ represent conventional BPSK modulated symbols transmitted in the 1st and 2nd time slots and no effort is made to jointly design a signal constellation for the various combinations of $x_1$ and $x_2$. For the sake of generalizing our treatment, let us assume that there are $L$ legitimate space-time signals $G_2(x_{i1}, x_{i2})$, $l = 0, 1, \ldots, L-1$, where $L$ represents the number of sphere-packed modulated symbols. The transmitter, then, has to choose the modulated signal from these $L$ legitimate symbols, which have to be transmitted over the two antennas in two consecutive time slots, where the throughput of the system is given by $(\log_2 L)/2$ bits per channel use. In contrast to Alamouti’s independent design of the two time slots’ signals, our aim is to design $x_{11}$ and $x_{12}$ jointly, such that they have the best minimum Euclidean
distance from all other \((L-1)\) legitimate transmitted space-time signals, since this minimizes the system’s error probability. Let \(\{a_{1,l}, a_{2,l}, a_{3,l}, a_{4,l}\}, l = 1, \ldots, L-1\), be phasor points from the four-dimensional real-valued Euclidean space \(\mathbb{R}^4\), where each of the four elements \(a_{1,l}, a_{2,l}, a_{3,l}, a_{4,l}\) gives one coordinate of the two time-slots’ complex-valued phasor points. Hence, \(x_{1,l}\) and \(x_{2,l}\) may be written as

\[
\{x_{1,l}, x_{2,l}\} = T(a_{1,l}, a_{2,l}, a_{3,l}, a_{4,l}) = \{a_{1,l} + ja_{2,l}, a_{3,l} + ja_{4,l}\},
\]

In the four-dimensional real-valued Euclidean space \(\mathbb{R}^4\), the lattice \(D_4\) is defined as a sphere packing having the best minimum Euclidean distance from all other \((L-1)\) legitimate constellation points in \(\mathbb{R}^4\) [50]. More specifically, \(D_4\) may be defined as a lattice that consists of all legitimate sphere packed constellation points having integer coordinates \([a_1, a_2, a_3, a_4]\) uniquely and unambiguously describing the legitimate combinations of the two time-slots’ modulated symbols in Alamouti’s scheme, but subjected to the sphere packing constraint of \(a_1 + a_2 + a_3 + a_4 = k\), where \(k\) is an even integer. Assuming that \(S = \{s' = [a_{1,l}, a_{2,l}, a_{3,l}, a_{4,l}] \in \mathbb{R}^4 : 0 \leq l \leq L-1\}\) constitutes a set of \(L\) legitimate constellation points from the lattice \(D_4\) having a total energy of \(E = \sum_{l=1}^{L} |a_{1,l}|^2 + |a_{2,l}|^2 + |a_{3,l}|^2 + |a_{4,l}|^2\), and upon introducing the notation

\[
C_l = \sqrt{\frac{2L}{E}} G_s(x_{1,l}, x_{2,l}), \quad l = 0,1,\ldots,L-1,
\]

we have a set of space-time signals, \(\{C_l : 0 \leq l \leq L-1\}\), whose diversity product is determined by the minimum Euclidean distance of the set of \(L\) legitimate constellation points in \(S\).

The following example illustrates how sphere packing modulation may be implemented in combination with \(G_s\)-space-time coded systems.

**Example:** Assume that there are \(L = 16\) different legitimate space-time signals, \(G_s(x_{1,l}, x_{2,l}), l = 0,1,\ldots,15\), that the encoder can choose from. We will consider two optional modulation schemes, namely conventional QPSK modulation and sphere packing modulation.

- **Conventional QPSK Modulation:**
  There are four legitimate two-bit QPSK symbols, \(S0, S1, S2,\) and \(S3\), that can be used for representing any of the symbols \(x_{1,l}\) and \(x_{2,l}\), \(l = 0,1,\ldots,15\). The transmission regime of the two consecutive time slots is demonstrated in Figure 18.

- **Sphere Packing Modulation:**
  We need \(L = 16\) phasor points selected from the lattice \(D_4, (a_{1,l}, a_{2,l}, a_{3,l}, a_{4,l}), l = 0,1,\ldots,15\), in order to jointly represent each space-time signal \((x_{1,l}, x_{2,l}), l = 0,1,\ldots,15\), according to Equation (11), as depicted in Figure 19. Table 5 shows the effective throughput and the associated transmission block sizes for different values of \(L\).
3.1.3 Sphere Packing Constellation Construction

Since the orthogonal $G_2$ space-time signal, which is constructed from the sphere packing scheme of Equation (12) is multiplied by a factor that is inversely proportional to $\sqrt{E}$, namely by $\frac{2L}{\sqrt{E}}$, it is desirable to choose a specific subset of $L$ points from the entire set of legitimate constellation points hosted by $D_*$, which results in the minimum total energy $E$ while maintaining a certain minimum distance amongst the SP symbols. Viewing this design trade-off from a different perspective, if more than $L$ points satisfy the minimum total energy constraint, an exhaustive computer search
<table>
<thead>
<tr>
<th>$L$</th>
<th>Block Size (bits)</th>
<th>Throughput (b/s/Hz)</th>
</tr>
</thead>
<tbody>
<tr>
<td>4</td>
<td>2</td>
<td>1</td>
</tr>
<tr>
<td>8</td>
<td>3</td>
<td>1.5</td>
</tr>
<tr>
<td>16</td>
<td>4</td>
<td>2</td>
</tr>
<tr>
<td>32</td>
<td>5</td>
<td>2.5</td>
</tr>
<tr>
<td>64</td>
<td>6</td>
<td>3</td>
</tr>
<tr>
<td>128</td>
<td>7</td>
<td>3.5</td>
</tr>
<tr>
<td>256</td>
<td>8</td>
<td>4</td>
</tr>
<tr>
<td>512</td>
<td>9</td>
<td>4.5</td>
</tr>
<tr>
<td>1024</td>
<td>10</td>
<td>5</td>
</tr>
<tr>
<td>2048</td>
<td>11</td>
<td>5.5</td>
</tr>
<tr>
<td>4096</td>
<td>12</td>
<td>6</td>
</tr>
</tbody>
</table>

Table 5 - Throughput of sphere packing aided $G_2$ systems for different SP signal set sizes $L$.

is carried out for determining the optimum choice of the $L$ points out of all possible points, which possess the highest MED, hence minimizing the SP-symbol error probability. For this purpose, the legitimate constellation points hosted by $D_4$ are categorized into layers or shells based on their norms or energy (i.e. distance from the origin) as seen in Table 6.

For example, it was shown in [50] that the first layer consists of 24 legitimate constellation points hosted by $D_4$ having an identical minimum energy of $E = 2$. In simple terms, the SP symbol centred at (0, 0, 0, 0) has 24 minimum-distance or closest-neighbour SP symbols around it, centred at the points (+/-1, +/-1, 0, 0), where any choice of signs and any ordering of the coordinates is legitimate.

Example: Assume that there are $L = 16$ different legitimate space-time signals, $G_2(x_1, x_2)$, $l = 0, 1, \ldots, 15$, which the encoder can choose from. Then, we need the most ‘meritorious’ $L = 16$ phasor points from the lattice $D_4$ for representing the ST-signals. According to the above-mentioned minimum energy constraint, only the 24 legitimate constellation points hosted by the first SP layer of Table 6 are considered, as shown in Figure 20. Then, an exhaustive computer search is employed for determining the optimum choice of the $L = 16$ points out of the 24 possible first-layer points, which possess the highest MED.

![Figure 20 - The 24 first-layer SP constellation points hosted by $D_4$ having the minimum energy of $E = 2$.](image)

Table 6 provides a summary of the constellation points hosted by the first 10 layers in the 4-dimensional lattice $D_4$. In order to generate the full list of SP regimes for a specific layer, we have to apply all legitimate permutations and signs for the corresponding constellation points given in Table 6.
### 3.1.4 STBC-SP Performance

In this section, a two transmit antenna based STBC-SP scheme is considered. Simulation results are provided for systems having different bits-per-symbol (BPS) throughputs in conjunction with the appropriate conventional and sphere packing modulation schemes, as outlined in Table 7. Observe that T=2 time slots are required for transmitting a single sphere packed symbol, when using the Nt=2 transmit antenna based STBC-SP scheme. By contrast, two conventionally modulated symbols are transmitted during the same time period. Therefore, the throughput of the sphere packing modulation scheme has to be twice that of the conventional modulation scheme in order to create systems having an identical overall BPS throughput. This explains the specific choices of L in Table 7. Results are also shown for systems employing Nr=1, 2, 3, 4, 5, and 6 receive antennas, when communicating over a correlated Rayleigh fading channel having a normalised Doppler frequency of fD = 0.1. The channel’s complex fading envelope is assumed to be constant over the transmission period of T=2 time slots of a single space-time signal or, equivalently, the transmission period of a sphere packed symbol. This type of channel will be referred to here as a sphere packing symbol invariant (SPSI) channel. In this section, both the achievable bit error rate (BER) and sphere packing symbol error rate (SP-SER) are considered. The SP-SER represents the block error rate, where the block size is B=\log_2 L bits, which is also synonymous to the space-time symbol error rate (ST-SER).

<table>
<thead>
<tr>
<th>Layer</th>
<th>Constellation Points</th>
<th>Norm</th>
<th>Number of Combinations</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0 0 0 0 0 0 1</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>+/-1 +/-1 0 0 2 24</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>+/-2 0 0 0 4 8</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>+/-1 +/-1 +/-1 +/-1 4 16</td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>+/-2 +/-1 +/-1 0 6 96</td>
<td></td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>+/-2 +/-2 0 0 8 24</td>
<td></td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>+/-2 +/-2 +/-1 +/-1 10 96</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>+/-3 +/-1 0 0 10 48</td>
<td></td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>+/-3 +/-1 +/-1 +/-1 12 64</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>+/-2 +/-2 +/-2 0 12 32</td>
<td></td>
<td></td>
</tr>
<tr>
<td>7</td>
<td>+/-3 +/-2 +/-1 0 14 192</td>
<td></td>
<td></td>
</tr>
<tr>
<td>8</td>
<td>+/-2 +/-2 +/-2 +/2 16 16</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>+/-4 0 0 0 16 8</td>
<td></td>
<td></td>
</tr>
<tr>
<td>9</td>
<td>+/-4 +/-1 +/-1 0 18 96</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>+/-3 +/-2 +/-2 +/-1 18 192</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>+/-3 +/-3 0 0 18 24</td>
<td></td>
<td></td>
</tr>
<tr>
<td>10</td>
<td>+/-4 +/-2 0 0 20 48</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>+/-3 +/-3 +/-1 +/-1 20 96</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 6 - The first 10 layers of $D_4$.

### Table 7 - Conventional and sphere packing modulation employed for different BPS throughputs.

<table>
<thead>
<tr>
<th>Throughput (BPS)</th>
<th>Conventional Modulation</th>
<th>Sphere Packing Modulation</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>BPSK</td>
<td>L=4</td>
</tr>
<tr>
<td>2</td>
<td>QPSK</td>
<td>L=16</td>
</tr>
<tr>
<td>3</td>
<td>8-PSK</td>
<td>L=64</td>
</tr>
<tr>
<td>4</td>
<td>16-QAM</td>
<td>L=256</td>
</tr>
</tbody>
</table>

Throughput (BPS) Conventional Modulation Sphere Packing Modulation

FP7 ICT Call 1 Key line 1.5 “Networked Media”
Sphere packing aided orthogonal design schemes promise to provide improved SP-SER (or ST-SER), when compared to the same metric of conventionally modulated orthogonal STBC based schemes. This promise is based on the fact that sphere packing modulation improves the diversity product (or coding advantage). On the other hand, the BER performance of sphere packing aided orthogonal ST-code design schemes is not always guaranteed to be better than that of conventionally modulated space-time signals, since sphere packing modulation specifically optimises the MED between two distinct space-time signals constructed using Equation (12), but not between the individual constituent symbols, $x_1, x_2, \ldots, x_{k+1}$, which conventional modulation based STBC optimises. However, it will be demonstrated later in this section that upon increasing the number of receive antennas, the achievable SP-SER performance improvement increases, which in turn leads to further BER performance improvements for the sphere packing aided orthogonal STBC schemes, as compared to conventionally modulated orthogonal STBC schemes.

Figure 21 shows the SP-SER performance curves of different orthogonal STBC schemes in combination with both conventional and sphere packing modulation for the different BPS throughputs, as outlined in Table 7. The systems employ $N_t=2$ transmit antennas and $N_r=1$ receive antenna for communicating over a correlated SPSI Rayleigh fading channel having a normalised Doppler frequency of $f_D=0.1$. It is seen from Figure 21 that for a particular BPS throughput, the two curves corresponding to both the conventional scheme and to the sphere packing modulation scheme have the same asymptotic slope (i.e. diversity level). This agrees with the observation stated in [51], namely that $G_2$-based space-time schemes dispensing with SP modulation also achieve full diversity. Therefore, SP is not expected to improve the asymptotic slope of the performance curves. Nonetheless, Figure 21 shows that orthogonal STBC using sphere packing offers a coding advantage over the conventionally modulated orthogonal STBC design. For example, sphere packing modulation having a throughput of 3 BPS and $L=64$ achieves a coding gain of about $1.2\text{dB}$ over classic 8-PSK modulated STBC at an SP-SER of $10^{-4}$. The corresponding BER performance curves are shown in Figure 22. The BER performances of sphere packing modulation and conventional modulation are identical for systems having rates of 1 and 2 BPS since it can be shown that QPSK for example constitutes an SP scheme. However, Figure 22 also shows that conventional modulation based STBC outperforms sphere packing modulation, when higher BPS throughputs are considered, when employing $N_r=1$ receive antenna. The performance results when using $N_r=2$ receive antennas are demonstrated in Figure 23 and Figure 24.

The coding gains achieved by sphere packing modulation over conventional modulation schemes at an SP-SER of $10^{-4}$ for the schemes are summarised in Table 8. Figure 25, Figure 26 and Figure 27 illustrate the SNR values required to achieve an SP-SER of $10^{-4}$ by the different orthogonal STBC schemes in combination with both conventional and sphere packing modulation for the different BPS throughputs of Table 7, when employing $N_r=2$ transmit and $N_r=1$, 3 and 6 receive antennas, respectively, for communicating over a correlated SPSI Rayleigh fading channel having a normalised Doppler frequency of $f_D=0.1$.

Observe in Figure 21 to Figure 24 that as alluded to before, the performance curves of QPSK modulation and sphere packing modulation having $L=16$ (i.e. 2 BPS schemes) are identical. This phenomenon is due to the fact that QPSK modulation constitutes a special case of sphere packing modulation, when it is combined with $G_2$ space-time signals. More specifically, consider the $G_2$ space-time signal defined as $G_2(x_{l1}, x_{l2})$, $l=0, \ldots, 15$. If $x_{l1}$ and $x_{l2}$ are chosen independently from the QPSK modulation constellation, then the 16 legitimate space-time signals produced will be identical to the 16 legitimate space-time signals constructed using Equation (11) and (12), where $(a_{l1}, a_{l2}, a_{l3}, a_{l4})$, $l=0, \ldots, 15$, correspond to the 16 SP constellation points hosted by $D_4$ that are centred at all possible permutations of $(+/1, +/1, +/1)$ and have a normalisation factor of $1/\sqrt{2}$. These SP constellation points belong to the second layer of $D_4$ seen in Table 6.

<table>
<thead>
<tr>
<th>$N_r$</th>
<th>1 BPS</th>
<th>2 BPS</th>
<th>3 BPS</th>
<th>4 BPS</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.4dB</td>
<td>0.0dB</td>
<td>1.2dB</td>
<td>0.6dB</td>
</tr>
<tr>
<td>2</td>
<td>0.5dB</td>
<td>0.0dB</td>
<td>1.6dB</td>
<td>0.6dB</td>
</tr>
<tr>
<td>3</td>
<td>0.6dB</td>
<td>0.0dB</td>
<td>2.0dB</td>
<td>0.9dB</td>
</tr>
<tr>
<td>4</td>
<td>0.4dB</td>
<td>0.0dB</td>
<td>1.9dB</td>
<td>0.7dB</td>
</tr>
<tr>
<td>5</td>
<td>0.5dB</td>
<td>0.0dB</td>
<td>1.9dB</td>
<td>0.9dB</td>
</tr>
<tr>
<td>6</td>
<td>0.4dB</td>
<td>0.0dB</td>
<td>2.1dB</td>
<td>0.9dB</td>
</tr>
</tbody>
</table>
Figure 21: Sphere packing symbol error rate of different orthogonal STBC schemes in combination with both conventional and sphere packing modulation for the different BPS throughputs of Table 7, when employing \( N_t=2 \) transmit and \( N_r=1 \) receive antennas for communicating over a correlated SPSI Rayleigh fading channel having a normalised Doppler frequency of \( f_D=0.1 \).

Figure 22: Bit error rate of different orthogonal STBC schemes in combination with both conventional and sphere packing modulation for the different BPS throughputs of Table 7, when employing \( N_t=2 \) transmit and \( N_r=1 \) receive antennas for communicating over a correlated SPSI Rayleigh fading channel having a normalised Doppler frequency of \( f_D=0.1 \).
Figure 23: Sphere packing symbol error rate of different orthogonal STBC schemes in combination with both conventional and sphere packing modulation for the different BPS throughputs of Table 7, when employing $N_t=2$ transmit and $N_r=2$ receive antennas for communicating over a correlated SPSI Rayleigh fading channel having a normalised Doppler frequency of $f_D=0.1$.

Figure 24: Bit error rate of different orthogonal STBC schemes in combination with both conventional and sphere packing modulation for the different BPS throughputs of Table 7, when employing $N_t=2$ transmit and $N_r=2$ receive antennas for communicating over a correlated SPSI Rayleigh fading channel having a normalised Doppler frequency of $f_D=0.1$. 
Figure 25: SNR required to achieve an SP-SER of $10^{-4}$ by different orthogonal STBC schemes in combination with both conventional and sphere packing modulation for the different BPS throughputs of Table 7, when employing $N_t=2$ transmit and $N_r=1$ receive antennas for communicating over a correlated SPSI Rayleigh fading channel having a normalised Doppler frequency of $f_D=0.1$.

Figure 26: SNR required to achieve an SP-SER of $10^{-4}$ by different orthogonal STBC schemes in combination with both conventional and sphere packing modulation for the different BPS throughputs of Table 7, when employing $N_t=2$ transmit and $N_r=3$ receive antennas for communicating over a correlated SPSI Rayleigh fading channel having a normalised Doppler frequency of $f_D=0.1$. 
Figure 27: SNR required to achieve an SP-SER of $10^{-4}$ by different orthogonal STBC schemes in combination with both conventional and sphere packing modulation for the different BPS throughputs of Table 7, when employing $N_t=2$ transmit and $N_r=6$ receive antennas for communicating over a correlated SPST Rayleigh fading channel having a normalised Doppler frequency of $f_D=0.1$.

### 3.2 Optimisation of Multidimensional BICM-ID Constellation Labelling

#### 3.2.1 Overview on Constellation Labelling

The BER performance curves of bit-interleaved coded modulation using iterative decoding (BICM-ID) may be divided into three regions: 1) the region of low signal-to-noise ratio (SNR) with negligible iterative BER reduction; 2) the so-called turbo-cliff or waterfall region exhibiting persistent iterative BER reduction over many iterations; and finally 3) the BER floor region typically experienced at moderate to high SNRs, where a rather low BER can be reached after a few iterations [54]. In order to attain a near-capacity performance, the SNR value corresponding to the turbo cliff has to be reduced. The crucial point at the receiver is that the concatenated detectors/demappers/decoders are soft-in/soft-out (SISO) decoders that accept and deliver probabilities or soft values, where the extrinsic information of the soft-output of one detector/demapper/decoder is passed on to the other detector/demapper/decoder to be used as a priori input.

Consequently, in [55], the employment of the turbo principle [56] was considered for iterative soft demapping in the context of BICM-ID, where a soft demapper was used between the multilevel demodulator and the channel decoder. Since then, it was widely recognised that the choice of a specific bit-to-symbol mapping scheme or a beneficial constellation labelling is an influential factor, when designing BICM-ID schemes exhibiting a high iteration gain [57][58][59][60][61]. In [57] ten Brink advocated the construction of different bit-to-symbol mapping schemes based on the bitwise mutual information, whereas the mapping schemes designed in [60] was based on the so-called Binary Switching Algorithm (BSA), which was previously employed for codebook index assignment optimisation in vector quantization [62]. Recently, it was shown in [61] that the constellation design problem may be viewed as Quadratic Assignment Problem (QAP) [63], which may be optimised using the powerful Reactive Tabu Search (RTS) technique of [64].

Generally, a larger constellation size of a higher-dimensional space renders the bit-to-symbol mapping design more flexible. Multi-dimensional constellations were shown to be beneficial in the design of TCM schemes as early as 1987 [65][66]. Additionally, multi-dimensional labelling was also proposed for QPSK based BICM-ID for transmission over
a single antenna [67]. Further improvements of multi-dimensional constellation labelling were proposed in [68] [69]. More recently, multi-dimensional constellation labelling was also proposed for bit-interleaved space-time-coded modulation using iterative decoding in [70], where the labelling of the two 16-QAM symbols transmitted over two antennas in two consecutive time-slots was designed jointly and was optimised using the RTS technique [64].

![Figure 28: The block diagram of a BICM-ID system.](image)

### 3.2.2 System Overview

The schematic of the entire system is shown in Figure 28, where the transmitted source bits $u_1$ are encoded by the channel encoder. This section considers a 1/2-rate recursive systematic convolutional (RSC) code. The channel encoded bits $c_1$ are then interleaved by a random bit interleaver, producing the permuted bits $u_2$. After channel interleaving, the $M$-ary modulator maps blocks of $n \cdot B$ channel-coded and interleaved bits to $n$ consecutive complex-valued $M$-ary symbols $s = [s_1, \ldots, s_n]$, where $B = \log_2 M$. There are $M^n$ distinct combinations of the $n$ consecutive complex-valued $M$-ary symbols that the modulator can choose from. The $n$ consecutive symbols can be represented in the $2n$-dimensional real-valued Euclidean space $\mathbb{R}^{2n}$ [71] as follows: $s = [s_1, \ldots, s_n] = [a_1, a_2, \ldots, a_{2n}]$, where $s_i = a_{2i-1} + ja_{2i}$ represents the $i$-th complex-valued $M$-ary symbol.

In this section, we consider an uncorrelated narrowband Rayleigh fading channel, based on Jakes’ fading model [72]. The complex-valued fading envelope $h$ is assumed to be different for each complex-valued $M$-ary symbol. The complex Additive White Gaussian Noise (AWGN) of $w = w_I + jw_Q$ is also added to the received signal, where $w_I$ and $w_Q$ are two independent zero-mean Gaussian random variables having a variance of $\sigma_w^2 = N_0/2$ per dimension, where $N_0/2$ represents the double-sided noise power spectral density expressed in $\text{W/Hz}$. Accordingly, the received signal corresponding to the $2n$-dimensional transmitted symbol $s = [s_1, \ldots, s_n]$ may be represented by $r = [r_1, \ldots, r_n]$, where $r_i = h_is_i + w_i$, $i = 1, \ldots, n$.

As shown in Figure 28, each $2n$-dimensional symbol $r$ represents a block of $n \cdot B$ coded bits. Assuming that the complex-valued fading envelope $h$ is perfectly known at the receiver, iterative demapping/decoding is carried out between the demodulator and the APP-based SISO channel decoder, where extrinsic information is exchanged between the constituent demapper/decoder modules [55][73]. More specifically, $L_{a}(\cdot)$ in Figure 28 represents the a priori information, expressed in terms of the log-likelihood ratios (LLRs) of the corresponding bits, whereas $L_{e}(\cdot)$ represents the extrinsic LLRs of the corresponding bits. The iterative process is performed for a number of consecutive iterations. During the last iteration, only the LLR values $L_{e}(u_i)$ of the original uncoded systematic information bits $u_i$ are required, which are passed to a hard decision decoder in order to determine the estimated transmitted source bits $\hat{u}_1$, as shown in Figure 28.

### 3.2.3 Optimisation of the Constellation Labelling

The optimisation of the constellation labelling is obtained by invoking the RTS algorithm [61][64] to minimise the following error-bound based cost function [60]
\[
CF_{eb} = \frac{1}{B^2} \sum_{i=1}^{B} \sum_{b=0}^{1} \sum_{s_i \in \mathcal{X}_i} \sum_{s_k \in \mathcal{X}_k} \frac{1}{|s_k - s_i|^2},
\]

(13)

where \( \mathcal{X}_i \) denotes the subset of symbols \( s_k \in \mathcal{X} \) whose bit labels have the value of \( b \in \{0,1\} \) in position \( i \in \{1, \ldots, B\} \). For example, consider a QPSK constellation consisting of four symbols, where each symbol represents two binary bits \((b_2, b_1)\) as follows: \( S_0 \mapsto 00 \), \( S_1 \mapsto 01 \), \( S_2 \mapsto 10 \) and \( S_3 \mapsto 11 \). Then, the cost function of Equation (13) becomes

\[
CF_{eb} = \frac{1}{8} \left( \frac{2}{|S_0 - S_1|^2} + \frac{2}{|S_0 - S_2|^2} + \frac{4}{|S_0 - S_3|^2} + \frac{4}{|S_1 - S_2|^2} + \frac{2}{|S_1 - S_3|^2} + \frac{2}{|S_2 - S_3|^2} \right),
\]

where the value of \( CF_{eb} \) clearly depends on the complex-valued representations of \( S_0 \), \( S_1 \), \( S_2 \) and \( S_3 \), which is basically the mapping scheme. Subsequently, the constellation labelling plays a crucial role in the optimisation of the cost function in Equation (13).

The optimisation problem in Equation (13) is similar to the facility-location problem in classical QAP [63]. More specifically, assuming that the constellation points as locations and the constellation labelling as facilities, the problem is to find the optimum facility-location pair that minimises the assignment cost function [61]

\[
\min_{\mu} \sum_{i=1}^{B} \sum_{b=0}^{1} \sum_{s_i \in \mathcal{X}_i} \sum_{s_k \in \mathcal{X}_k} \frac{1}{|s_k - s_i|^2},
\]

(14)

where \( \mu \) is the labelling rule between the facilities and locations. The QAP formulation of Equation (14) may be expressed as [61]

\[
\min_{\mu} \sum_{i=1}^{M} \sum_{j=1}^{M} f_{ij} d_{\mu(i), \mu(j)},
\]

(15)

where we define the distance between any two locations \( s_i \) and \( s_j \) as [61]

\[
d_{s_i, s_j} = \begin{cases} 
|s_i - s_j|^2, & i \neq j \\
0, & i = j
\end{cases}
\]

and define the flow between constellation point’s label \( i \) and \( j \) as [61]

\[
f_{ij} = \begin{cases} 
1, & i \text{ and } j \text{ differ only in one bit position} \\
0, & \text{otherwise}
\end{cases}
\]

Equation (15) may be efficiently solved using the RTS algorithm [64].
3.2.4 EXIT Chart Analysis

In this section, we present the EXIT chart analysis and results of several conventional and multidimensional BICM-ID schemes, where the constellation labelling was optimized according to the method discussed in Section 3.2.3. The objective of this section also is to demonstrate the benefits of multidimensional BICM-ID schemes.

The main objective of employing EXIT charts [57], is to predict the convergence behaviour of the iterative decoder by examining the evolution of the input/output mutual information exchange between the inner and outer decoders in consecutive iterations. The application of EXIT charts is based on the two assumptions, namely that upon assuming large interleaver lengths, (1) the *a priori* LLR values are fairly uncorrelated; (2) the *a priori* LLR values exhibit a Gaussian PDF.

Let $I_{\cdot,a}(x), 0 \leq I_{\cdot,a}(x) \leq 1$, denotes the mutual information between the *a priori* LLRs $L_{\cdot,a}(x)$ as well as the corresponding bits $x$ and let $I_{\cdot,e}(x), 0 \leq I_{\cdot,e}(x) \leq 1$, denote the mutual information between the extrinsic LLRs $L_{\cdot,e}(x)$ and the corresponding bits $x$, where the subscript $(\cdot)$ is used to distinguish the channel decoder ($D$) and the demodulator ($M$).

Let $I_{\cdot,a}(x), 0 \leq I_{\cdot,a}(x) \leq 1$, denote the mutual information between the *a priori* LLRs $L_{\cdot,a}(x)$ as well as the corresponding bits $x$ and let $I_{\cdot,e}(x), 0 \leq I_{\cdot,e}(x) \leq 1$, denote the mutual information between the extrinsic LLRs $L_{\cdot,e}(x)$ and the corresponding bits $x$, where the subscript $(\cdot)$ is used to distinguish the channel decoder ($D$) and the demodulator ($M$).

The exchange of extrinsic information in the system schematic of Figure 28 is visualised by plotting the extrinsic information transfer characteristics of the demodulator and of the outer RSC decoder in a joint diagram. This diagram is known as the Extrinsic Information Transfer (EXIT) chart [57]. The outer RSC decoder’s extrinsic output $I_{D,e}(c_1)$ becomes the demodulator’s *a priori* input $I_{M,a}(u_2)$, which is represented on the $x$-axis. Similarly, on the $y$-axis, the demodulator’s extrinsic output $I_{M,e}(u_2)$ becomes the outer RSC decoder’s *a priori* input $I_{D,a}(c_1)$. For the rest of this section $I_{D,\cdot}(c_1)$ and $I_{M,\cdot}(u_2)$ will be simply denoted by $I_{D,\cdot}$ and $I_{M,\cdot}$, respectively.

Figure 29 shows the EXIT chart of a BICM-ID scheme employing QPSK modulation and a 1/2-rate RSC code having octally represented generator polynomials of $(G_r,G) = (7,5)_8$, where $G_r$ is the feedback polynomial. Figure 29 illustrates the EXIT chart of the system, when operating at $E_b/N_0=5$dB and communicating over the uncorrelated Rayleigh fading channel considered. Ideally, in order for the exchange of extrinsic information between the demodulator and the outer RSC decoder to converge at a specific $E_b/N_0$ value, the EXIT curve of the demodulator recorded at the $E_b/N_0$ value of interest and that of the outer RSC decoder should only intersect at a point infinitesimally close to the $I_{D,e} = 1.0$ line. If this condition is satisfied, then a so-called *convergence tunnel* [57] appears in the EXIT chart. The narrower the tunnel, the more iterations are required for reaching the point of intersection.

![EXIT chart of a QPSK-modulated BICM-ID scheme employing Gray mapping (GM) and anti-Gray mapping (AGM) in combination with a 1/2-rate RSC code, when communicating over an uncorrelated Rayleigh fading channel.](image)

Figure 29: EXIT chart of a QPSK-modulated BICM-ID scheme employing Gray mapping (GM) and anti-Gray mapping (AGM) in combination with a 1/2-rate RSC code, when communicating over an uncorrelated Rayleigh fading channel.
As expected, Gray mapping (GM) does not provide any iteration gain upon increasing the mutual information at the input of the demodulator [54]. However, using anti-Gray mapping (AGM) schemes [55] results in beneficial iteration gain, upon increasing the mutual information, as illustrated by the positive slopes of the EXIT curves of the AGM schemes seen in Figure 29. Any bit-to-symbol mapping scheme, which is different from classic Gray mapping is termed as an AGM scheme, as detailed in [54].

Observe in also Figure 29 that when detecting two QPSK symbols jointly, the attainable iteration gain achieved upon increasing the mutual information becomes higher than in case of considering each QPSK symbol separately. The higher iteration gain is represented by the steeper slope of the EXIT curve in Figure 29. This implies that according to the predictions of the EXIT chart seen in Figure 29, the iterative decoding process is expected to converge to a lower BER at $E_b/N_0 = 5.0\text{dB}$, when considering a pair of QPSK symbols jointly, as compared to the separate decoding and demapping of the QPSK symbols.

### 3.2.5 BER Performance

These EXIT chart based convergence predictions are verified by the actual BER performance depicted in Figure 30, which compares the performance of the BICM-ID system employing GM and AGM schemes in conjunction with QPSK modulation and the RSC code of Figure 29, when communicating over an uncorrelated Rayleigh fading channel and employing an interleaver depth of $D = 10^5$ bits. Observe in Figure 30 by comparing the two GM-based BICM-ID curves that as expected no BER improvement was obtained, when five turbo-detection iterations were employed in conjunction with GM. This phenomenon was reported also in [55] and becomes evident from the horizontal curve of the GM seen in Figure 29. By contrast, AGM achieved a useful performance improvement in conjunction with iterative demapping and decoding. Moreover, as predicted from the EXIT chart of Figure 29, the joint demapping and decoding of two QPSK symbols provided further BER performance improvements compared to the separate demapping and decoding of QPSK symbols.

![Figure 30: Performance comparison of the BICM-ID system employing GM and AGM schemes in conjunction with QPSK modulation and a 1/2-rate RSC code, when communicating over an uncorrelated Rayleigh fading channel and employing an interleaver depth of $D=10^5$ bits.](image-url)
3.3 Multicell Cooperation Based SVD Assisted Multi-User MIMO Transmission

3.3.1 Introduction

Multiple-input multiple-output (MIMO) systems are capable of supporting high-rate, high-integrity transmission [74]. Intensive research efforts have been dedicated to single-user MIMO (SU-MIMO) designs [75][88]. For multi-user MIMO (MU-MIMO) systems, spatial-division multiple-access (SDMA) has been proposed, where each user's unique channel state information (CSI) is used to distinguish them. Furthermore, in order to simplify the receiver's design at the mobile station (MS) in the context of downlink (DL) transmissions, transmit pre-processing has been proposed to move the part of the required signal processing from the MS to the base station (BS) [76][77][78][79]. In the simple single-cell scenario only intra-cell interference has been considered [77][78], but owing to frequency reuse, inter-cell interference is imposed by the other cells [80].

Recently, multiple base station (BS) aided systems have attracted substantial attention [81][82][83][84][85]. It has been shown that for a multiple cell system, the achievable system performance can be substantially improved, if cooperative BSs are invoked [81][82]. For the DL of cooperative BSs, various joint transmission schemes have been proposed. The attainable system capacity was approached by the so-called dirty paper (DP) precoding [81]. However, due to its complexity, linear joint transmission schemes may be more preferred, such as joint zero-forcing (ZF), joint minimum mean square error (MMSE) based DL processing, joint block diagonalization (BD), and joint signal to leakage plus noise ratio (SLNR) based processing [83][84].

For the cooperative BS aided DL system, each MS may be able to synchronously receive its own signal from different BSs. However, the interference is essentially asynchronous and ignoring this asynchronous interference may result in a severe performance loss [83][84]. Fortunately, the joint ZF and joint BD based transmission schemes are capable of completely eliminating the interference, which is in contrast to the residual interference experienced after joint MMSE or SLNR processing.

However, the joint ZF based transmission results in several parallel single input single output (SISO) channels, which results in a performance loss compared to the joint detection of the transmitted symbols. Joint BD based transmission is capable of generating several parallel single user multiple input multiple output (SU-MIMO) systems, facilitating the joint DL detection of parallel streams in the transmitted MIMO symbols. However, a specific drawback of the joint BD based transmission is that the resultant SU-MIMO channel will have to be estimated by each MS [86], which may impose an excessive complexity. Recently, singular value decomposition (SVD) based multiuser transmission has been developed in the literature for single-cell systems, which is also capable of decomposing the MU-MIMO system into parallel SU-MIMO systems, enabling the joint detection of the transmitted SU-MIMO symbols [87]. Naturally, the effective SU-MIMO channel is entirely determined by each MS's own channel, hence no extra training or overhead is needed compared to BD based transmission.

In this section, we extend the SVD-based MU-MIMO transmission from the single-cell case to the multicell scenario. Furthermore, the achievable system performance is evaluated in conjunction with two different power allocation schemes derived for both the BD, SVD and ZF based multicell transmission schemes.

![Figure 31: System Overview of multicell cooperation aided transmission](image-url)
Let us assume that there are \( K \) co-channel mobile users arbitrarily distributed in the DL of a multicell system, with \( N_k \) being the number of receive antennas at each MS and \( N_b \) the number of adjacent co-channel BSs in the system, with \( N_k \) being the number of transmit antennas at each BS, respectively as in Figure 31 for a scenario associated with \( N_k = K = 3 \).

Assuming non-dispersive or flat-fading conditions, let \( H_{j,k} \) be the small-scale fading channel matrix characterizing the channel between BS \( j \) to MS \( k \) having zero-mean, unit-variance complex-Gaussian entries, and let \( a_{j,k} \) be the corresponding large-scale fading coefficients including the effect of both path-loss and shadow fading.

### 3.3.2 SVD Based Multicell Transmission

Let us assume that the \( N_k \)-component DL transmitted signal vector destined for the \( k \)-th MS is given by \( x_k \) and that in the case of BS cooperation, we pre-process \( x_k \) according to

\[
\begin{align*}
    d_k &= P_k \beta_k x_k, \\
    &= d_k P_k \beta_k
\end{align*}
\]

where \( P_k \) is an \( M \times N_k \)-element pre-processing matrix with \( M = N_k \times N_k \), which is responsible for cancelling the multiuser interference (MUI), hence resulting in an effective SU-MIMO system. Furthermore, \( \beta_k \) is a \( N_k \times N_k \)-element diagonal matrix hosting the power control coefficients.

The signal transmitted from all \( N_b \) BSs to all \( K \) MSs can be expressed as

\[
    d = \sum_{k=1}^{K} d_k = \sum_{k=1}^{K} P_k \beta_k x_k = P \beta x,
\]

where we have

\[
    P = \begin{bmatrix} P_1 & P_2 & \cdots & P_K \end{bmatrix},
\]

\[
    \beta = \text{diag} \left[ \beta_1, \beta_2, \cdots, \beta_K \right],
\]

and \( x \) is a \( \sum_{k=1}^{K} N_k \)-component vector containing the transmitted data, which is given by

\[
    x = \begin{bmatrix} x_1^T, x_2^T, \cdots, x_K^T \end{bmatrix}.
\]

Now the signal vector \( y_k \) received at the \( k \)-th MS from all \( N_b \) BSs can be expressed as

\[
    y_k = H_k d + n_k,
\]

where the channel matrix \( H_k \) is given by

\[
    H_k = [a_{1,k} H_{1,k}, a_{2,k} H_{1,k}, \cdots, a_{K,k} H_{1,k}],
\]

and \( n_k \) represents the \( N_k \)-element AWGN vector having a zero mean and a covariance matrix of \( \mathbb{E}[n_k n_k^H] = \sigma^2 I_{N_k} \). Furthermore, the all signal vector \( y \) received by all the \( K \) MSs from all \( N_b \) BSs can be expressed as

\[
    y = \begin{bmatrix} y_1^T, y_2^T, \cdots, y_K^T \end{bmatrix}^T = \begin{bmatrix} H_1^T, H_2^T, \cdots, H_K^T \end{bmatrix} d + n.
\]

Let us assume that the rows of \( H_k \) have full rank, i.e. we have \( \text{rank}(H_k) = N_k \) and that \( M = \sum_{i=1}^{K} N_i \) is satisfied. Then, upon carrying out the SVD of \( H_k \), we arrive at
\[ H_k = U_k \left[ A_k^{1/2}, 0 \right] V_k^H = U_k \begin{bmatrix} A_k^{1/2} V_{k_{in}}^H \\ V_k^H \end{bmatrix} = U_k A_k^{1/2} V_k^H, \]  

(24)

where \( U_k \) and \( V_k \) are \( N_k \times N_k \)-component and \( M \times M \)-component unitary matrices, respectively. Furthermore, \( A_k \) is a \( N_k \times N_k \)-component diagonal matrix containing the eigenvalues of \( H_k H_k^H \), i.e. we have \( A_k = \text{diag}[\lambda_{k1}, \lambda_{k2}, \cdots, \lambda_{kN_k}] \). Finally, \( V_{k_{in}} \) in Equation (24) is an \( M \times N_k \)-component matrix, which is constituted by the eigenvectors corresponding to the non-zero eigenvalues of \( H_k H_k^H \). By contrast, \( V_{k_{in}} \) in Equation (24) is an \( M \times (M - N_k) \)-component matrix, which is constituted by the eigenvectors corresponding to the zero eigenvalues of \( H_k H_k^H \). Similarly, \( U_k \) consists of the eigenvectors of \( H_k H_k^H \).

Upon substituting Equation (24) into Equation (23), we arrive at

\[ y = U A U^H P x + n, \]  

(25)

where we have

\[ U = \text{diag}[U_{k1}, U_{k2}, \cdots, U_k], \]
\[ A = \text{diag}[A_{k1}, A_{k2}, \cdots, A_k], \]
\[ V_{k_{in}} = \begin{bmatrix} V_{k_{in1}} & V_{k_{in2}} & \cdots & V_{k_{inM}} \end{bmatrix}, \]
\[ n = \begin{bmatrix} n_{k1} & n_{k2} & \cdots & n_k \end{bmatrix}^T. \]

(26)

In Equation (26) \( U \) and \( A \) are \( \sum_{i=1}^{K} N_i \times \sum_{i=1}^{K} N_i \)-component matrices, \( V_{k_{in}} \) is a \( M \times \sum_{i=1}^{K} N_i \)-component matrix and \( n \) is an AWGN vector, which is Gaussian distributed with zero-mean and a covariance matrix of \( \sigma^2 I_{N_k} \).

For SVD based pre-processing, the pre-processing matrix \( P \) can be set to be [87]

\[ P = \begin{bmatrix} V_{k_{in1}}^H \\ \vdots \\ V_{k_{inM}}^H \end{bmatrix} V_{k_{in}} \]  

(27)

where \( \left[ V_{k_{in}}^H \right] \) denotes the pseudo-inverse of the matrix \( \left[ V_{k_{in}} \right] \). Consequently, the resultant signal received at the \( k \)-th MS is given by

\[ y_k = U_k A_k^{1/2} P_{k_{in}} x_k + n_k, \quad k = 1, 2, \cdots, K. \]  

(28)

3.3.3 The Achievable Maximum Transmission Rate

The major difference between multi-cell and single-cell transmission is that the power constraints have to be considered on a per-BS basis, i.e. the average transmit power of the \( j \)-th BS is limited by \( P_j \).

For multicell transmission, the pre-processing matrix for the \( k \)-th MS can be expressed as

\[ P_{k_{in}} = \begin{bmatrix} P_{k1}^T & P_{k2}^T & \cdots & P_{kK}^T \end{bmatrix}, \]  

(29)

where the \( N_i \times N_k \)-dimensional matrix \( P_{k_{in}} \) is the pre-processing matrix configured for transmission from the \( j \)-th BS to the \( k \)-th MS.
Furthermore, let the $N \times \sum_{k=1}^{K} N_k$-dimensional matrix $\overline{P}_j$ denote the pre-processing matrix configured for transmission from the $j$th BS to all the $K$ MSs, which is given by

$$\overline{P}_j = \begin{bmatrix} P_{j,1} & P_{j,2} & \cdots & P_{j,K} \end{bmatrix},$$

which represents the pre-processing matrix configured for transmission from the $j$th BS to all the $K$ MSs.

In order to meet the per-BS power constraints, we have to satisfy

$$\text{Tr}[\overline{P}_j^H \overline{P}_j] = \text{Tr}[\beta\overline{P}_j^H \overline{P}_j] = \sum_{i=1}^{K} \beta_i^2 \left[ \overline{P}_j^H \overline{P}_j \right]_{ii} \leq P_j,$$

where $[\overline{P}_j^H \overline{P}_j]_{ii}$ is the $i$th diagonal element of the matrix $[\overline{P}_j^H \overline{P}_j]$ and $\text{Tr}[\bullet]$ denotes the trace of the argument.

The maximum achievable average transmission rate per user per antenna is given by

$$R_{\text{SYB}} = \max \left\{ \frac{1}{\sum_{k=1}^{K} N_k} \sum_{i=1}^{K} \log_2 \left( 1 + \frac{1}{\sigma^2} \lambda_k \beta_{ji}^2 \right) \right\},$$

under the constraints of Equation (31).

The above optimization problem may be solved for example, by the interior-point method as recommended in [85]. However, since it can be complex to deal with, we resort to a less complex suboptimal solution in this section.

### 3.3.3.1 Scaled Suboptimal Power Allocation

First, the so-called scaled power allocation is considered [85]. In this case, the power allocation is performed firstly by assuming that all BSs can jointly pool their power, i.e. the maximum achievable average rate is obtained by

$$R_{\text{joint}} = \frac{1}{\sum_{k=1}^{K} N_k} \sum_{i=1}^{K} \log_2 \left( 1 + \frac{1}{\sigma^2} \lambda_k \beta_{ji}^2 \right),$$

under the constraint of

$$\text{Tr}[\beta \overline{P}_j^H \overline{P}_j] = \sum_{i=1}^{K} \beta_i^2 \left[ \overline{P}_j^H \overline{P}_j \right]_{ii} \leq \sum_{j=1}^{N} P_j.$$

Then, the power allocation matrix is scaled by a factor of $\mu$, which is given by

$$\mu = \min_{j=1,2,\ldots,N} \frac{P_j}{\sum_{i=1}^{K} \beta_i^2 \left[ \overline{P}_j^H \overline{P}_j \right]_{ii}}.$$

Therefore, the maximum achievable average rate of this scheme is obtained by

$$R_{\text{scaled}} = \frac{1}{\sum_{k=1}^{K} N_k} \sum_{i=1}^{K} \log_2 \left( 1 + \mu \frac{1}{\sigma^2} \lambda_k \beta_{ji}^2 \right).$$

### 3.3.3.2 Grouped Suboptimal Power Allocation

Another suboptimal power allocation policy is to divide the transmitted symbol vector into $N_b$ groups, where each symbol of the same group is assigned the same power coefficient $u_j$, and the power vector $u = [u_1, \cdots, u_{N_b}]^T$ assigned to all the MSs can be expressed as [83].

$$u = Q^T \mathbf{p},$$

where $Q$ is an $N_s \times N_b$-element matrix given by
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\[ Q = \begin{bmatrix}
\text{Trace}(P_{1,1}^H P_{1,1}) & \cdots & \text{Trace}(P_{1,N_j}^H P_{1,N_j}) \\
\text{Trace}(P_{2,1}^H P_{2,1}) & \cdots & \text{Trace}(P_{2,N_j}^H P_{2,N_j}) \\
\vdots & \ddots & \vdots \\
\text{Trace}(P_{N_i,1}^H P_{N_i,1}) & \cdots & \text{Trace}(P_{N_i,N_j}^H P_{N_i,N_j})
\end{bmatrix}, \quad (38) \]

where \( P_{i,j} \) is the precoding matrix configured for DL transmission at the \( i \) th BS to the \( j \) th group of transmitted data. In the scenario where no feasible solution exists, all the symbols are assigned the same power \( \beta \), which is given by [83]

\[ \beta^2 = \min_{i=1,2,\ldots,N_i} \sum_{j=1}^{N_j} \frac{P_j}{P_j^H P_j}. \quad (39) \]

3.3.4 Simulation Results

![Figure 32: Average Capacity versus Average SNR performance for BD, SVD and ZF based Multicell Transmission for the case of three BSs and three MSs](image)

![Figure 33: Average Capacity versus average SNR performance for BD, SVD and ZF based multicell transmission for the case of two BSs and two MSs](image)
In this section, simulation results are presented to characterize the achievable performance of SVD-based multicell transmission in conjunction with the different power allocation schemes considered. The results are also compared to those of BD and ZF based multicell transmission schemes. Specifically, the large-scale fading is assumed to be $a_{j,k}^2 = 1$ for $j = k$, otherwise $a_{j,k}^2 = 0.5$ as in [83], which ignores the shadow fading. Furthermore, the transmission power of each BS is assumed to be the same, given by $P_t$ and the SNR is defined as $P_t \sigma^2$.

In Figure 32, the maximum achievable average capacity of BD, SVD and ZF based multicell transmission is investigated, where we assume that there are three BSs and three MSSs. Each of the BSs supports a single MS within its coverage area. Furthermore, the number of BS transmit antennas is assumed to be $N_t = 2$ and the number of receive antennas at the MS is $N_r = 2$. Furthermore, the performance of joint power allocation scheme of Equation (34) is also plotted as an upper bound. As we can see from Figure 32, the scaled suboptimal power allocation scheme of Equation (37) outperforms the grouped power allocation scheme of Equation (38) for any of the transmission strategies considered. Furthermore, the BD based scheme achieves the best performance, since it has a unitary pre-processing matrix [83], which does not reduce the effective power of each symbol. However, in order to generate the resultant SU-MIMO system CSI for each MS, extra training or overhead is invoked for the BD based scheme [86]. Moreover, the SVD based scheme has an approximately 1dB to 1.5dB performance gain over the ZF based scheme, since SVD based scheme results in joint reception of the transmitted SU-MIMO streams, while the ZF based scheme results in parallel SISO channels. Similar trends can also be observed in Figure 33, where we assume that there are two BSs and two MSSs, where each of the BSs has a single MS within its coverage area. Furthermore, the number of BS transmit antennas is assumed to be $N_t = 2$ and the number of receive antennas at the MS is $N_r = 2$.

In this section, we investigated the extension of SVD based multiuser transmission from the single cell case to the multicell scenario, where the multiple BSs may cooperate during their concerted transmissions. We also compared the SVD based BS cooperation to the corresponding BD and ZF based schemes, while using different power allocation policies. The BD based scheme is capable of achieving the best performance, while the SVD based scheme is suboptimal. However, the BD based scheme requires extra training for generating the effective SU-MIMO CSI, while the SVD based scheme avoids this requirement. Hence, the SVD based scheme is less complex than the BD based one.

### 3.4 P-STC for cooperative relaying: elements on code design and some preliminary results

Relaying and Cooperation have become an important research topic for wireless applications. In fact, IEEE 802.16 working group has devoted a task group to include relay capabilities into WiMAX standard. This task group is currently managing to finish the 802.16j Multihop Relay specification to expand 802.16e-2005 capabilities. This 16j specification will address only Point-to Multipoint OFDMA physical layer mode of WiMAX, by only requiring an upgrade of Base stations to support relaying. The benefits of relaying may be of two types: increase in capacity when relays cooperate with base station to send the same data to mobile station; increase in coverage when the relay route data from base stations which can not be heard from mobile stations. Both modes are supported in the specification. The first mode, from the transmission point of view, aims at increase diversity in the form of cooperative diversity: with cooperative transmission space-time codes can be utilized as distributed codes across antennas at the cooperating relays and base station.

Besides the interest that relaying technologies attract in 4G and WiMAX applications, these cooperative solutions also offer new degrees of freedom to be exploited for the adaptation and optimization of wireless multimedia communications, which is the main goal of the OPTIMIX project. The use of a simple flexible transmission scheme based on distributed space-time convolutional codes allows us to easily explore adaptive modulation/coding strategies jointly with an adaptive exploitation of the new air interface resources (i.e. the relays) provided by the wireless network infrastructure.

In this section we provide some elements for the design of simple scalable space time coding scheme for the implementation of the cooperative communication concept in the downlink of a wireless communication system. More precisely, we consider here a pragmatic approach based on the concatenation of convolutional codes and BPSK/QPSK modulation to obtain cooperative codes for relay networks, for which we derive the pairwise error probability, an asymptotic bound for frame error probability, and a design criterion to optimize both diversity and coding gain. Based on this framework, we set up a code search procedure to obtain a set of good pragmatic space-time codes (P-STCs) with overlay construction, suitable for cooperative communication with a variable number of relays in quasistatic channel, which outperform in terms of coding gain other space-time codes (STCs) proposed in the literature. We also find that, despite the fact that the implementation of pragmatic space-time codes requires standard
 convolutional encoders and Viterbi decoders with suitable generators and branch metric, thus having low complexity, they perform quite well in block fading channels, including quasistatic channel, even with a low number of states and relays.

3.4.1 System Model

The cooperative scheme is depicted in Figure 34 and follows time-division channel allocations with orthogonal cooperative diversity transmission. Each user (i.e., the source) divides its own time-slot into two equal segments, the first from time $t_1$ to $t_1 + \Delta$ and the second from $t_2 = t_1 + \Delta$ to $t_2 + \Delta$, where $\Delta$ is the segment duration. In the first segment, the source broadcasts its coded symbols; in the second all the active relays which are able to decode the message forward the information through proper encoding trying to take advantage of the overall available diversity. Thus, the design of proper STCs for the two phases is crucial to maximize both achievable diversity and coding gain.

We assume $n$ transmitting antennas at each terminal and $m$ receiving antennas at the destination. Hence, $n_s = n$ antennas will be used in the first phase and a total of $n_t = Rn$ antennas will be used in the second phase, where $R$ is the number of relays able to decode and forward the source message.

![Figure 34: Two-phase relaying scheme: phase 1 (continuous line), phase 2 (dashed line). Source, relays, and destination nodes are denoted with $S$, $R$, $D$, respectively.](image)

We indicate with $c^o_i$, the modulation symbol transmitted by relay $r$ ($0 \leq r \leq R$, and $r = 0$ is the source) on the antenna $i$ at discrete time $t$, that is, at the $t$th instant of the encoder clock. Each symbol is assumed to have unit norm and to be generated according to the modulation format by suitable mapping. Note that symbol $c^o_i$, is transmitted at time $t_i + t$, while symbols $c^o_r$, for $r > 0$ are transmitted at time $t_i + t$. The received signals corresponding to all symbols $c(t,r)$ are jointly processed by the decoder at the reference time $t$. We also denote with $C^n = [c^o_{1,1}, c^o_{1,2}, \ldots, c^o_{n,n}]$ a supersymbol, which is the vector of the $(R + 1)n$ outputs of the overall “virtual encoder” constituted by the source encoder and the relays’ encoders. A codeword is a sequence $C = (C^n, \ldots, C^n)$ of $N$ supersymbols generated by the source and relays’ encoders. This codeword $C$ is interleaved before transmission to obtain the sequence $c_i = \Pi = (C_{\sigma_i,1}, \ldots, C_{\sigma_i,N})$, where $\sigma_1, \ldots, \sigma_i$ is a permutation of the integers $1, \ldots, N$, and $\Pi(\cdot)$ is the interleaving function.

The channel model includes additive white Gaussian noise (AWGN) and multiplicative flat fading, with Rayleigh distributed amplitudes assumed constant over blocks of $B$ consecutive transmitted space-time symbols and independent from block to block. Perfect channel state information is assumed at the decoder for each node. The transmitted supersymbol at time $\sigma_i$ goes through a compound channel described by the $((m + n) \times m)$ channel matrix $H^{\sigma_i} = [H^{\sigma_i,1}, \ldots, H^{\sigma_i,m}]$, where $H^{\sigma,i} = [h^{\sigma,i}_{r,s}]$, and $h^{\sigma,i}_{r,s}$ is the channel gain between transmitting antenna $i$, with $i = 1, \ldots, n$, of the terminal $r$ and receiving antenna $s$, with $s = 1, \ldots, m$, at time $\sigma_i$. In the BFC model, these channel matrices do not change for $B$ consecutive transmissions, thus we actually have only $L = N/B$ possible distinct channel matrix instances per codeword (for the sake of simplicity, we assume that $N$ and $B$ are such that $L$ is an integer). Denoting by $Z = \{Z_s, \ldots, Z_s\}$ the set of $L$ channel instances, we have

$$H^{\sigma_i} = Z_s$$

for $\sigma = (l - 1)B + 1, \ldots, lB, \quad l = 1, \ldots, L$.

When the fading block length, $B$, is equal to one, we have the ideally interleaved fading channel (i.e., independent fading levels from symbol to symbol), while for $L = 1$, we have the quasi-static fading channel (fading level constant over a codeword); by varying $L$, we can describe channels with different correlation degrees.

At the receiving side, the sequence of received signal vectors is $r_t = (R^{o_1}, \ldots, R^{o_N})$, and after deinterleaving we have $r = \Pi^t(r_t) = (R^{o_1}, \ldots, R^{o_N})$, where the received vector at time $t$ is $R^o = [r^{o_{1,1}}, r^{o_{1,2}}, \ldots, r^{o_{m,m}}]$ with components

$$r^{(l,1)}_s = \sqrt{E_s} \sum_{i=1}^n h^{(l,1)}_{s,0,i} c^{(l,1)}_i + \eta^{(l,1)}_s, \quad s = 1, \ldots, m,$$

in the first phase and
\[ r_s^{[l,2]} = \sqrt{E \sum_{r=1}^{K} \sum_{i=1}^{n} h_{r,i}^{[s]} r_{r,i}^{[l,2]} + n_s^{[l,2]}}, \quad s = 1, \ldots, m, \]
in the second phase. In this equation, \( r_s^{[s]} \) is the signal space representation of the signal received by antenna \( s \) at time \( l \) in phase \( l \), the noise terms \( n_s^{[s]} \) are independent, identically distributed (i.i.d.) complex Gaussian random variables (r.v.s), with zero mean and variance \( \text{No}^2/2 \) per dimension, and the r.v.s \( h_{r,i}^{[s]} \) represent the deinterleaved complex Gaussian fading coefficients. Since we assume spatially uncorrelated channels, these are i.i.d. with zero mean and variance 1/2 per dimension, and consequently \( |h_{r,i}^{[s]}| \) are Rayleigh distributed r.v.s with unit power. The constellations are multiplied by a factor \( \sqrt{E} \) in order to have a transmitted energy per symbol equal to \( E_s \), which is also the average received symbol energy. The total energy transmitted per supersymbol is \( E_r = (n+\text{No})E \) and the energy transmitted per information bit is \( E_i = E_s/hR_s \), where \( h \) is the number of bits per modulation symbol and \( R_s \) is the overall code rate of the cooperative space-time code. Thus with ideal pulse shaping, the spectral efficiency is \( nhR_s/2 \) [bps/Hz].

For following discussions sections, it is worthwhile to recall that, over a Rayleigh fading channel, the system achieves a diversity \( D \) if the asymptotic error probability is \( P_e = K(E_s/\text{No})^D \), where \( K \) is a constant depending on the asymptotic coding gain. In other words, a system with diversity \( D \) is described by a curve of error probability with a slope approaching \( 10/D \) [dB/decade] for large signal-to-noise ratio (SNR).

### 3.4.2 Pragmatic space-time codes for cooperative relaying

In the case of the two-phase relaying scheme shown in Figure 34, the probability of transmission failure over the two phases depends on the number of relays available for cooperation and on the link qualities on source-destination, source-relays, and relays-destination. In the applications of interest the set of relays is initialized at the beginning of a data communication session and is kept unchanged over a long period of many slots. The set of relays is chosen by looking at active terminals able to guarantee a good average link quality (depending on terminal position and slow fading) with the source terminal. During this period, a cooperative coding scheme is used by the source and the set of relays to protect the transmission of data frames between source and destination. Sometimes, due to fast fading fluctuations, it may happen that one or more relays are not able to decode the source codewords in phase 1. In the simplified case of equal quality on all source-relay links, denoting by \( P_e^{(S-R)} \) the error probability for the link from source to destination, \( P_e^{(S-R)} \) the error probability for the source-relay link (i.e., \( P_e^{(S-R)} = \cdots = P_e^{(S-R)} = P_e^{(S-R)} \)), and with \( P_e^{(SR-R)} \) the error probability for the link from the source plus \( k \) relays to destination, the overall error probability \( P_e \) is given by

\[
P_e = \left( P_e^{(S-R)} \right)^R P_e^{(S-D)} + \sum_{k=1}^{R} \binom{R}{k} \left( P_e^{(S-R)} \right)^{R-k} \left( 1 - P_e^{(S-R)} \right)^k P_e^{(SR-Rk-D)}.
\]

For the design of the coding scheme with cooperative relays, it is generally recognized that the code components used by the source in phase 1 should maximize diversity and coding gain for each link connecting the source to relays and destination. The other code components should be designed to maximize diversity and coding gain of the entire cooperative code, that is, the code including all the code components transmitted during phases 1 and 2, for any possible number of cooperative relays.

In our work, we are considering the design of space-time trellis codes for relaying networks by using the pragmatic approach. Our proposed “pragmatic” approach uses a low-complexity architecture for STCs where the code components are built by the concatenation of a binary convolutional encoder and binary phase shift keying (BPSK) or quaternary phase shift keying (QPSK) modulator. Our “pragmatic” approach thus consists in using common convolutional codes as space-time codes, with the architecture presented in Figure 35. Here, \( k \) information bits are encoded by a convolutional encoder with rate \( k/\text{Nh} \). The \( nh \) output bits are divided into \( n \) streams, one for each transmitting antenna, of BPSK \((h = 1)\) or QPSK \((h = 2)\) symbols that are obtained from a natural (Gray) mapping of \( h \) bits. By natural mapping; we mean that for BPSK an information bit \( b \in \{0, 1\} \) is mapped into the antipodal symbol \( c = 2b - 1 \), giving \( c \in \{-1, +1\} \); for QPSK a pair of information bits, \( a, b \), is mapped into a complex symbol \( c = (2a-1)\sqrt{2} + j(2b-1)/\sqrt{2}, \) giving \( c \in \{\pm 1/\sqrt{2} \pm j/\sqrt{2} \} \), with \( j = \sqrt{-1} \). Then, each stream of symbols is eventually interleaved. If \( \mu \) is the encoder constraint length, then the associated trellis has \( N_\mu = 2^{2H-1} \) states.
Figure 35: Architecture of pragmatic space-time codes. For cooperative P-STCs, the “distributed” convolutional encoder is the ensemble of $R + 1$ single encoders, one for each transmitter; hence, instead of $n$, we must consider the overall number of antennas.

We can describe P-STCs for cooperative communication, obtained by joining the $R + 1$ code components used by the cooperating transmitters, by using the trellis of each encoder (the same as for the convolutional codes (CC)), labelling the generic branch from state $S_i$ to state $S_j$ with the supersymbol $C_{S_i ightarrow S_j} = [c_0, \ldots, c_R]_t^f$, where for BPSK, the symbol $c_{r,i}$ is the output (in antipodal form) of the $i$th generator of the $r$th transmitter. One of the advantages of the pragmatic architecture is that the maximum likelihood (ML) decoder is the same Viterbi decoder of the convolutional encoder adopted in transmission (same trellis), with a simple modification of the branch metrics. Being $\{c_{r,i}\}$ the set of output symbols labelling the branch, the branch metric for the Viterbi decoder is thus given by

$$P_2 = \frac{1}{4N_0} \sum_{j=1}^{m} \left[ \left| r_{j,1}^{(1,1)} - \sqrt{E_s} \sum_{i=1}^{n} h_{0,j,i}^{(0)} c_{0,i} \right|^2 + \left| r_{j,2}^{(1,2)} - \sqrt{E_s} \sum_{r=1}^{R} \sum_{i=1}^{n} h_{r,j,i}^{(r)} c_{r,i} \right|^2 \right].$$

P- Thus, the advantages of P-STCs with respect to STCs are as follows: the encoder is a common convolutional encoder;
(ii) the (Viterbi) decoder is the same as for a convolutional code, except for a change in the metric evaluation;
(iii) P-STCs are easy to study and optimize, even over BFC.
These advantages apply also when P-STCs are used for cooperative communications, as it will be further investigated in the next sections.

3.4.3 Performance analysis for cooperative space-time codes over BFC
We first consider the derivation of the pairwise error probability (PEP). Given the transmitted codeword $c$ and another codeword $g \neq c$, the PEP, that is, the probability that the ML decoder favours the codeword $g$ over $c$, conditional to the set of fading levels $Z$, can be written as

$$P_2 \{ \xi \rightarrow g \mid Z \} = \frac{1}{2} \text{erfc} \left( \frac{E_s}{\sqrt{4N_0}} d^2(\xi, g \mid Z) \right),$$

where erfc$(x)$ is the complementary error function, and the conditional Euclidean squared distance is given by

$$d^2(\xi, g \mid Z) = \sum_{r=1}^{N} \sum_{s=1}^{m} \left[ \sum_{i=1}^{n} h_{r,i,s}^{(r)} (c_{r,i} - g_{r,i}) \right]^2.$$
where

\[ A^{(l)}(\xi, \gamma) = \begin{bmatrix} a^{(l,1)} & 0 \\ 0 & a^{(l,2)} \end{bmatrix}, \]

\[ a^{(l,1)} = \left( c^{(l,1)} - g^{(l,1)} \right) \left( c^{(l,1)} - g^{(l,1)} \right)^H, \]

\[ a^{(l,2)} = \left( c^{(l,2)} - g^{(l,2)} \right) \left( c^{(l,2)} - g^{(l,2)} \right)^H. \]

after having split the generic supersymbol \( C(t) \) in the two parts transmitted during phase 1 and phase 2, respectively, that is, \( C^{(l)} = [c^{(l,1)} \ c^{(l,2)}]^T \), where \( c^{(l,1)} = [c_{0,1}, \ldots, c_{0,d}] \) and \( c^{(l,2)} = [c_{1,1}, \ldots, c_{d,1}] \). Due to the BFC assumption, for each frame and each receiving antenna, the fading channel is described by only \( L \) different vectors \( h^{(l)} \in \{ h^{(1)} \}, h^{(2)}, \ldots, h^{(L)} \), \( s = 1, \ldots, m \), where \( z_{s,t} \) is the \( s \)th row of \( Z_{t} \). By grouping these vectors, we can rewrite the squared distance as

\[ d^2(\xi, \gamma | Z) = \sum_{l=1}^{L} \sum_{s=1}^{m} z_{s,l}^2 \left( H^{(l)}(\xi, \gamma) Z_{s,l}^H \right), \]

and \( \mathcal{T}(l) = \{ t : H^{(a)} = Z_{t} \} \) is the set of indexes \( t \) where the channel fading gain matrix is equal to \( Z_{t} \). This set depends on the interleaving strategy adopted. Note that in our scheme (Figure 35), the interleaving is done “horizontally” for each transmitting antenna and in the same way for each transmitter, and that the set \( \mathcal{T}(l) \) is independent of \( s \), in other words, that the interleaving rule is the same for all antennas.

The matrix \( F^{(c, g)} \) is also Hermitian nonnegative definite, being the sum of Hermitian nonnegative definite matrices. It has, therefore, real nonnegative eigenvalues. Moreover, it can be written as \( F^{(c, g)} = U(l)A(l)U(l)^H \), where \( U(l) \) is a unitary matrix and \( A(l) \) is a real diagonal matrix, whose diagonal elements \( \lambda_i(l) \) with \( i = 1, \ldots, n = n_1 + n_2 \) are the eigenvalues of \( F^{(c, g)} \) counting multiplicity. As a result, we can express the squared distance in terms of the eigenvalues of \( F^{(c, g)} \) as follows:

\[ d^2(\xi, \gamma | Z) = \sum_{l=1}^{L} \sum_{s=1}^{m} z_{s,l}^2 \left( H^{(l)}(\xi, \gamma) Z_{s,l}^H \right) = \sum_{l=1}^{L} \sum_{s=1}^{m} \sum_{i=1}^{n} \lambda_i(l) | \beta_{s,i}^l |^2, \]

where \( \beta_{s,i}^l = [\beta_{1,1}^l, \beta_{1,2}^l, \ldots, \beta_{d,1}^l] = z_{s,l} U(l)^H \).

It should be observed that the form of matrix \( F^{(c, g)} \) is different from the matrix of the same space-time code working on a system with \( n_1 + n_2 \) transmit antennas due to the use of two distinct transmission phases in the cooperative system. Therefore, the same code used in the cooperative system may achieve different diversity and coding gains. It should also be noted that this matrix is diagonal (hence full-rank) only when \( n = 1 \) and \( R = 1 \). When transmitters have more than one antenna or more than one relay cooperate to transmission, only a suitable choice of the code may lead to a full-rank matrix, as shown later.

Vector \( z_s(l) \) has independent, complex Gaussian elements, with zero mean and variance \( 1/(2\sigma^2) \) for each transmit antenna due to the use of two distinct transmission phases in the cooperative system.

The integer \( \eta(l) \) has the same statistical description of \( \eta \). Moreover, for BFC, vectors \( z_s(l) \) and \( z_s(l) \) are independent for all \( l \neq j \). Hence, the unconditional pairwise error probability (PEP) becomes

\[ P(\xi_g) = \frac{1}{2} \text{erfc} \left( \sqrt{\frac{E_s}{4N_0} \sum_{s=1}^{m} \sum_{i=1}^{\eta(l)} | \beta_{s,i}^l |^2} \right), \]

where \( \text{erfc} \) indicates expectation with respect to fading (i.e., over the distribution of the \( \beta_{s,i}^l \)). By evaluating the asymptotic behaviour for large SNR, we obtain:

\[ P(\xi_g) \leq K(m\eta) \left[ \frac{E_s}{4N_0} \sum_{l=1}^{L} \prod_{i=1}^{\eta(l)} | \beta_{s,i}^l |^2 \right]^{-m}, \]

where \( K(d) = 1/(2d-1)(2d-1)!/(d!(d-1)!) \). The integer \( \eta(l) \) is the number of nonzero eigenvalues of \( F^{(c, g)} \), and \( \eta \) (which we call the pairwise transmit diversity) is the sum of the ranks of \( F^{(c, g)} \), for \( l = 1, \ldots, L \). The PEP between \( c \) and \( g \) shows a diversity \( m\eta \), that is, the product of transmit and receive diversity.

Therefore, it is clear that the performance analysis of a cooperative space-time codes with fixed number of cooperating relays is similar to the analysis of common space-time codes as in [88][89]. The only difference lies in the structure of the matrix \( F^{(c, g)} \) which has some zero off diagonal elements and may therefore have different rank and eigenvalues.
where $P(c)$ is the probability of transmitting the codeword $c$ (i.e., for P-STCs, equal to $2^{-kN}$ for equiprobable input bit sequence and $2^{-k(N+\mu)}$ for a zero-tailed code) and $I(c)$ is the codeword error probability for the transmitted codeword $c$. By using the asymptotic approximation and by observing that the retained dominant terms are those with transmit diversity $\eta_{min} = \min_{c}(\eta(c))$, where $\eta(c) = \min_{g}(\eta(g, c))$ and $E(c, x) = \{g \neq c : \eta(c, g) = x\}$ is the set of codeword sequences at minimum diversity, the asymptotic error probability bound can be written as

$$\hat{P}_{w} = \sum_{\xi} P(\xi) P_{w}(\xi) \leq \sum_{\xi} \sum_{\xi \neq \xi} P(\xi) P(\xi \rightarrow \xi')$$

From (21), we observe that the asymptotic performance of STCs in BFC depends on both the achievable diversity, $\eta_{min}$, and the performance factor

$$\hat{P}_{min}(m) = \sum_{\xi} P(\xi) F_{min}(\xi, m) \triangleq \sum_{\xi} P(\xi) \sum_{g \in E(\xi, \eta_{min})} \left( \prod_{i=1}^{L} \eta_{i}^{(l)} \right)^{-m}$$

This performance factor is related to the coding gain. Note also that $\eta_{min}$ does not depend on the number of receiving antennas. Therefore, when a code is found to reach the maximum diversity $\eta_{min}$ in a system with one receiving antenna, the same code reaches the maximum diversity $\eta_{min} m$ when used with multiple receiving antennas. However, due to the presence of the exponent $m$ in each term of the last sum, the best code (i.e., the code having the smallest performance factor) for a given number of antennas is not necessarily the best for a different number of receiving antennas. Thus, a search for optimum codes in terms of both diversity and performance factor must in principle be pursued for each $m$.

To summarize, the derivation of the asymptotic behaviour of a given STC with a given length requires the computation of the matrices $F^{(i)}(c, g)$ with their rank and product of nonzero eigenvalues. Moreover, according to [90], by restricting in the bound the set of sequences $g$ to those corresponding to paths in the trellis diagram of the code diverging only once from the path of codeword $c$, the union bound becomes tighter and can be evaluated in an effective way, by using the methodology illustrated in [89] through the concept of space-time generalized transfer function.

### 3.4.4 Pragmatic space-time code design for relaying

In this section, we address the issues of how to set up design criteria for good cooperative STCs and how to perform an efficient search for the optimum (in a sense defined later) generators for the code components of cooperative STCs in BFC. In general the design of good cooperative STCs may be based on one of the following approaches.

- By assuming that the cooperative code is working with a predefined number of cooperative relays $R$, it may be designed as P-STC with $k$ binary inputs and $n(R + 1)$ output symbols which maximize diversity gain and coding gain. A pragmatic suboptimal solution to this problem may be to build the code using the rate $k/nh(R + 1)$ maximum free distance convolutional code, optimum for the AWGN channel. This design method does not guarantee that the first rate $k/nh$ component code used in phase 1 is the best performing code. It also does not guarantee good performance when some code components are not used by relays unable in some frames to decode the source message. Moreover, the pragmatic solution may be not optimal even in terms of diversity gain and therefore should be checked by means of simulations. However, we observed that in many cases this solution leads to quite good results.

- By assuming that the cooperative code is obtained by joining code components in phase 2 from every relay able to decode the source message, the code may be designed as STC with overlay construction. With this method, a good code for $R$ relays is designed starting from a good code for $R – 1$ relays and by adding the best code component that maximizes diversity and coding gain of the final code. In this way, the first code component used by the source in phase 1 is always a good code. In the case of a fixed set of cooperating relays, the sequence of additional code components can be assigned to the relays ranked in order of average link quality in such a way that the second code component is assigned to the relay with the best link quality and so on, thus they are used with high probability in the same combinations for which they have been designed. Moreover, it is easier to design the additional code components than the entire cooperative code.

We propose here to set up an STC code search that aims at seeking cooperative codes covering both the outlined design approaches, that is, the design of an entire rate $k/nh(R + 1)$ P-STCs, and the design of rate $k/nh$ code components in an overlay structure. The search criterion proposed here is based on the asymptotic error probability, so that the optimum code with fixed parameters $(n, k, h, \mu)$, among the set of noncatastrophic codes, is the code that

- (i) maximizes the achieved diversity $\eta_{min}$;
The transmitters determine the first code component considers the best rate achievable on the reference BFC, is bounded by the maximum diversity per receiving antenna offered by the channel, \( R(n) \). It is not achievable that only a related to the Singleton bound for BFC. Let us define the reference block fading channel (RBFC) for the system as the ideal equivalent BFC with the codes among those achieving the same diversity \( \tilde{\eta}_{\min} \).

Of course, the achievable diversity is the most important design parameter. Since \( \tilde{\eta}_{\min} \) cannot be larger than \( \eta(c, g) \leq (R+1)nL \) and the free distance \( d_f \) of the convolutional code used to build the P-STCs, it appears that to capture the maximum diversity per receiving antenna offered by the channel, \((R+1)nL\), the free distance of a good code for a given BFC should be at least \((R+1)nL\) or larger. On the other hand, there is a fundamental limit on the achievable diversity related to the Singleton bound for BFC. Let us define the reference block fading channel (RBFC) for the system as the ideal equivalent BFC with \((R+1)nL\) fading blocks that would describe the space-time fading channel if the \((R+1)n\) transmitters determine \((R+1)n\) independent channels. The achievable diversity, which cannot be larger than the diversity achievable on the reference BFC, is bounded by

\[
\tilde{\eta}_{\min} \leq 1 + \left[ L(R + 1)n \left( 1 - \frac{k}{(R+1)nh} \right) \right].
\]

As an example, to achieve full diversity \((R + 1)n\) with PSTCs in a quasistatic channel \((L = 1)\), the value \(k/h\) cannot be larger than 1, thus the code rate of each convolutional code component cannot be larger than \(1/n\), or the value of \(h\) cannot be smaller than \(k\).

### 3.4.5 Results

In this section, we report the results obtained in our search for good cooperative STCs with overlay construction for different system configurations with \( R = 1 \) relay, \( n = 2 \) transmitting antennas, \( m = 1 \) receiving antennas. All the codes proposed are full-diversity codes. Two approaches are considered for overlay construction: the first considers the use of the maximum free distance (optimum for AWGNN channel) code \( k/hn \) code as the first code component; the second as the first code component considers the best rate \( k/hn \) P-STCs reported in [89].

#### Table 9: Optimum overlays for rate \(1/(2R)\) COP-STC with BPSK, \(n=2, m=1, R=1\), in quasistatic channel. The basic code for a single transmitter is STC as in [89].

<table>
<thead>
<tr>
<th>( \mu )</th>
<th>Generators ( r = 0 )</th>
<th>Generators ( r = 1 )</th>
<th>( F_{\min}^{(1)}/N )</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>((1,2)_8)</td>
<td>((1,2)_8)</td>
<td>0.0044</td>
</tr>
<tr>
<td>3</td>
<td>((3,4)_8)</td>
<td>((5,7)_8)</td>
<td>0.0015</td>
</tr>
<tr>
<td>4</td>
<td>((13,15)_8)</td>
<td>((11,17)_8)</td>
<td>0.0008</td>
</tr>
<tr>
<td>5</td>
<td>((23,31)_8)</td>
<td>((27,35)_8)</td>
<td>0.0006</td>
</tr>
</tbody>
</table>

Suboptimal cooperative STCs, working with a predefined number of cooperative relays \( R \) and constructed by pragmatically choosing the best (maximum distance) convolutional codes for AWGNN, can be easily obtained by using the convolutional code generators reported in [47]. It has been checked that this easy approach leads in most cases to acceptable results. However, it sometimes leads to cooperative STCs not achieving full diversity. As an example, this is the case of the rate \(1/4\), 4-state code with generators \((5, 7, 7, 7)_8\) for systems with BPSK, \(n=2\) and \(R=1\) cooperative relays, which achieves a maximum diversity of 3.

We also report the performance results, in terms of frame error rate (FER) as a function of SNR, for cooperative pragmatic space-time codes (CP-STCs). The SNR is defined as \( E_b/N_0 \) per receiving-antenna element where, for a fair comparison among situations with different number of relays, \( Eb \) is the total energy per information bit over all transmitting nodes and averaged with respect to fading. We refer here to applications with a static set of relays in the simplified case of equal quality on all source-relay links. The probability that a relay cooperates with the source is given by \( P_{\text{coop}} = 1 - P^{\text{all}-R} \). We first investigate in Figure 36 the effect of the number of states (ranging from 4 to 64) on the achievable diversity, in case of two relays cooperating with the source and BPSK modulation. It is noticeable that only a
portion of the available space-time diversity can be achieved depending on the number of states, that is, \( \eta_{\text{min}} = 3 \) for 4-state codes, \( \eta_{\text{min}} = 4 \) for 8 and 64 states, \( \eta = 5 \) for 16 and 32 states, respectively. Note also that the generators for 64 state codes achieve a diversity smaller than that for 32 state codes. This is due to the fact that pragmatic suboptimal construction does not always lead to the best possible generators for the direct and the relay phases. It is also worth observing that in the absence of relaying, this code is not able to capture the same diversity degree as with relaying (achieving diversity 2). In Figure 37, we show the FER versus SNR for CP-STCs with BPSK modulation, optimal generators for AWGN, 8 states. Here, the probability of cooperation \( P_{\text{coop}} \) takes values 0% (no cooperation), 50%, 90%, 95%, 99%, and 100% (i.e., certainty of cooperation). We can note that to approach the best performance a probability of cooperation larger than 0.9 is needed. On the other hand, the code used for 2 relays only achieves diversity 4.

![Figure 36: BPSK, optimal generators for AWGN, 4 to 64 states, 2 relays, 2 transmitting antennas per node, 1 receiving antenna, quasistatic fading channel L=1.](image)

![Figure 37: FER versus SNR for BPSK modulation, optimal generators for AWGN, 8 states, two relays, 2 transmitting antennas per node, one receiving antenna, respectively, in quasistatic fading channel (L=1).](image)
4 OMNeT++ modules

4.1 LDPC codes for erasure channels
OMNeT++ Codec modules for a family of packet erasure LDPC codes with selectable code rate (whose performance is reported in Figure 14) will be provided for the first version of the OMNeT++ simulation chain. The parameters for the codec module will be represented by erasure code symbol size, erasure code source block number, number of symbols per source block \((k)\), number of repair symbols to be generated (through the value of code rate \(R\)).

4.2 Elements on the Physical layer in the Base Station and the Mobile terminals
In Figure 38 and Figure 39 we show the graphical representation of the OMNeT++ NEtwork Description (NED) files of the BS and the generic mobile client considered within the first version of the OPTIMIX simulator. The layered structure of the video data flow is quite evident, as well as the control and feedback information exchanges between the different layers and the BS controller. In this paragraph we show in more details the internal architecture of the physical layer block (indicated in the figures with \(phy\)), highlighting its flexibility and its capability to represent the different channel coding/modulation/multiple access techniques considered in the project.

Figure 39 shows the structure of the mobile terminal unit: in addition to a \(phy\) block performing operations dual to those at the transmitter side, a radio channel module has been included.

![Figure 38](image_url)
In Figure 40 we report the internal structure of the phy block, mainly composed by:
- the channelCodec block,
- the mimoModem,
- the frameDeAssembler.

4.2.1 The channel Co-Decoder module
The input of the channel encoder module is a set of one or more packets, all selected from a single buffer in the data link layer. Based on its optimization strategy, the BS controller chooses the channel code type and the relevant
parameters to be applied to that particular input. According to the channel code selected and the length of the input sequence, the input bits may be either divided into multiple information words with fixed length (process sometimes indicated as slicing) or not. The slicing typically occurs with block channel codes (e.g. LDPCC) while direct encoding can be applied with codes supporting variable length information words (e.g. convolutional codes). A proper interleaver may follow the channel encoding process. The output of the channel co-decoder module is constituted by the succession of encoded and interleaved words.

The main control information provided by the BS controller are:
- the code type (e.g. none, LDPC C, CC, etc.);
- the code rate (selected as the index in a predefined set of possible values);
- additional code parameters, in case more codes belonging to the same group are available (e.g. constraint length for CC’s).

At the receiver side, the dual operations of deinterleaving and decoding will be performed starting from the LLR’s provided by the MIMO Demodulator module.

In the first version of the OPTIMIX simulator, the rate-compatible LDPC IRA codes described in section 2.3 and the corresponding slicing functionality will be included. At the receiver side we always assume to receive ideal control information about the selected code type, the code rate, etc. The implemented decoding process will be based on the Belief Propagation iterative algorithm.

4.2.2 The MIMO Mo-Demodulator module

At the transmitter side, the input of the modulator are the set of codewords provided by the channel encoder, while the output is constituted by a sequence of complex vectors, representing the symbols transmitted by the multiple antennas at the BS.

The main control information provided by the BS controller are:
- the MIMO mo-demodulator type (e.g. single or multiple beamforming, STC, etc.);
- the average transmission power;
- the constellation size (e.g. BPSK, QPSK, 16-QAM, 64QAM, etc.);
- the number of transmitting antennas;
- the number of receiving antennas;
- additional (coded) modulation parameter, in case more solutions belonging to the same family are available.

Dually, at the receiver side the sequence of complex vectors output from the radio channel module becomes the input for the demodulator module, which provides as output the LLR’s for the following channel decoder unit. As for the channel decoder, we assume to receive ideal control information about the selected modulator type, constellation size, etc. Clearly, at the receiver we always assume to have the CSI knowledge necessary to perform the selected demodulation process.

In the first version of the OPTIMIX simulator, the channel estimation at the receiver side will be considered as ideal. Depending on the particular communication scenario simulated, proper feedback algorithms are supposed to provide complete CSI at the transmitter or just some synthetic description of it.

4.2.3 OFDM Frame De-Assembler

The main goal of the Frame Assembler module at the transmitter side is to build the multi-user/multi-carrier/multi-antenna frames to be transmitted by the BS. In particular, the complex vectors coming from the MIMO modulator are allocated to the different Resource Units (RU) available in the OFDMA frames (each RU is constituted by a couple (subcarrier slot, time slot)), according to the information provided by the BS controller. When each OFDMA frame is complete, it is sent to the radio channel module. Different multiple access techniques can be easily simulated with this approach, like TDMA, FDMA and OFDMA.

The main control information provided by the BS controller to this module are:
- the destination user;
- the time-frequency position of the modulated complex vectors.

At the receiver side, dual operations are performed by the Frame Deassembler module: in particular, each user checks if the received frame contains data for him/her. If so, the module extracts the correspondent complex symbols from the frame and provides them to the demodulator.
In the OPTIMIX simulator we will always consider ideal carrier synchronization and symbol timing and information about the structure of the received frames will be always assumed as correctly received by the mobile terminals. Moreover, in the first version of the OPTIMIX simulator, the Frame De-Assembler module will support only a TDMA strategy.

4.2.4 Radio Channel Module

In the first version of the OPTIMIX simulator, we will consider a radio channel module taking into account block fading in both time and the frequency domain (see [7], section 8.1). Thus, its main configuration parameters will be the coherence time (i.e. the block length in the time domain), the bandwidth coherence (i.e. the block length in the frequency domain), the SNR, the number of transmitting/receiving antennas, etc.
5 Conclusion

The work on channel coding and modulation reported in this deliverable will continue throughout the lifetime of WP2 and will be presented in two more subsequent deliverables, namely D2.2b and D2.2c. Novel channel coding and modulation techniques as well as their pros and cons were presented. The state-of-the-art channel coding and modulation schemes proposed will be analysed and investigated further with respect to their application in the OPTIMIX simulation chain based on OMNeT++ and the OPTIMIX demonstrator.
6 References and glossary

6.1 References


6.2 Glossary

<table>
<thead>
<tr>
<th>Abbreviation</th>
<th>Description</th>
</tr>
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<tbody>
<tr>
<td>AGM</td>
<td>Anti-Gray Mapping</td>
</tr>
<tr>
<td>APP</td>
<td>A-posteriori probability</td>
</tr>
<tr>
<td>ARA</td>
<td>Accumulate-Repeat-Accumulate</td>
</tr>
<tr>
<td>ARQ</td>
<td>Automatic Repeat Request</td>
</tr>
<tr>
<td>AWGN</td>
<td>Additive White Gaussian Noise</td>
</tr>
<tr>
<td>BD</td>
<td>Block Diagonalization</td>
</tr>
<tr>
<td>BEC</td>
<td>Binary erasure channel</td>
</tr>
<tr>
<td>Acronym</td>
<td>Description</td>
</tr>
<tr>
<td>---------</td>
<td>-------------</td>
</tr>
<tr>
<td>BER</td>
<td>Bit Error Ratio</td>
</tr>
<tr>
<td>BFC</td>
<td>Block fading channel</td>
</tr>
<tr>
<td>BICM-ID</td>
<td>Bit-Interleaved Coded Modulation Using Iterative Decoding</td>
</tr>
<tr>
<td>BPSK</td>
<td>Binary Phase Shift Keying</td>
</tr>
<tr>
<td>BS</td>
<td>Base Station</td>
</tr>
<tr>
<td>BSA</td>
<td>Switching Algorithm</td>
</tr>
<tr>
<td>CCSDS</td>
<td>Consultative Committee For Space Data Systems</td>
</tr>
<tr>
<td>CER</td>
<td>Codeword error rate</td>
</tr>
<tr>
<td>CRC</td>
<td>Error Detection Code</td>
</tr>
<tr>
<td>CSI</td>
<td>Channel State Information</td>
</tr>
<tr>
<td>DL</td>
<td>Downlink</td>
</tr>
<tr>
<td>DP</td>
<td>Dirty Paper</td>
</tr>
<tr>
<td>ESI</td>
<td>Encoded Symbol ID</td>
</tr>
<tr>
<td>EXIT</td>
<td>Extrinsic Information Transfer</td>
</tr>
<tr>
<td>FEC</td>
<td>Forward error correction</td>
</tr>
<tr>
<td>FER</td>
<td>Frame error rate</td>
</tr>
<tr>
<td>fps</td>
<td>Frames-Per-Second</td>
</tr>
<tr>
<td>GE</td>
<td>Gaussian Elimination</td>
</tr>
<tr>
<td>GeIRA</td>
<td>Generalized irregular repeat-accumulate</td>
</tr>
<tr>
<td>GM</td>
<td>Gray Mapping</td>
</tr>
<tr>
<td>IRA</td>
<td>Irregular repeat-accumulate</td>
</tr>
<tr>
<td>ISCD</td>
<td>Iterative Source-Channel Decoding</td>
</tr>
<tr>
<td>JSCC</td>
<td>Joint Source and Channel Coding</td>
</tr>
<tr>
<td>LDGM</td>
<td>Low Density Generator Matrix</td>
</tr>
<tr>
<td>LDPC</td>
<td>Low-Density Parity-Check</td>
</tr>
<tr>
<td>LLR</td>
<td>Log-likelihood ratio</td>
</tr>
<tr>
<td>LT</td>
<td>Luby-Transform</td>
</tr>
<tr>
<td>MAP</td>
<td>Maximum A Posteriori</td>
</tr>
<tr>
<td>MBMS</td>
<td>Multimedia Broadcast Multicast Service</td>
</tr>
<tr>
<td>MBMS</td>
<td>Broadcast Multicast Service</td>
</tr>
<tr>
<td>MBs</td>
<td>Macro-Blocks</td>
</tr>
<tr>
<td>MED</td>
<td>Minimum Euclidean Distance</td>
</tr>
<tr>
<td>MIMO</td>
<td>Multiple Input Multiple Output</td>
</tr>
<tr>
<td>ML</td>
<td>Maximum Likelihood</td>
</tr>
<tr>
<td>MMSE</td>
<td>Minimum Mean Square Error</td>
</tr>
<tr>
<td>MPE</td>
<td>Multi Protocol Encapsulation</td>
</tr>
<tr>
<td>MS</td>
<td>Mobile Station</td>
</tr>
<tr>
<td>MU</td>
<td>Multi-User</td>
</tr>
<tr>
<td>MUI</td>
<td>Multi-user Interference</td>
</tr>
<tr>
<td>PSK</td>
<td>Phase Shift Keying</td>
</tr>
<tr>
<td>P-STC</td>
<td>Pragmatic space-time code</td>
</tr>
<tr>
<td>QAM</td>
<td>Quadrature Amplitude Modulation</td>
</tr>
<tr>
<td>QAP</td>
<td>Quadratic Assignment Problem</td>
</tr>
<tr>
<td>QCIF</td>
<td>Quarter Common Intermediate Format</td>
</tr>
<tr>
<td>QPSK</td>
<td>Quadrature Phase Shift Keying</td>
</tr>
<tr>
<td>RCC</td>
<td>Recursive Convolutional Code</td>
</tr>
<tr>
<td>RSC</td>
<td>Recursive Systematic Convolution</td>
</tr>
<tr>
<td>RTS</td>
<td>Reactive Tabu Search</td>
</tr>
<tr>
<td>SBCs</td>
<td>Short Block Codes</td>
</tr>
<tr>
<td>SBSD</td>
<td>Soft-Bit Source Decoding</td>
</tr>
<tr>
<td>SDMA</td>
<td>Spatial Division Multiple Access</td>
</tr>
<tr>
<td>SIRA</td>
<td>Systematic irregular repeat-accumulate</td>
</tr>
<tr>
<td>SISO</td>
<td>Single Input Single Output</td>
</tr>
<tr>
<td>SLNR</td>
<td>Signal To Leakage Plus Noise Ratio</td>
</tr>
<tr>
<td>SP</td>
<td>Sphere Packing</td>
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<td>-------</td>
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<tr>
<td>SP-SER</td>
<td>Sphere Packing Symbol Error Rate</td>
</tr>
<tr>
<td>STBC</td>
<td>Space-Time Block Coded</td>
</tr>
<tr>
<td>STBC</td>
<td>Space-Time Block Code</td>
</tr>
<tr>
<td>ST-SER</td>
<td>Space-Time Symbol Error Rate</td>
</tr>
<tr>
<td>SU</td>
<td>Single-User</td>
</tr>
<tr>
<td>SVD</td>
<td>Singular Value Decomposition</td>
</tr>
<tr>
<td>UEP</td>
<td>Unequal error protection</td>
</tr>
<tr>
<td>VNs</td>
<td>Variable Nodes</td>
</tr>
<tr>
<td>WLAN</td>
<td>Wireless local area network</td>
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<tr>
<td>ZF</td>
<td>Zero-Forcing</td>
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</tbody>
</table>

WiFi is a commercial name for a WLAN realization.