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Effects of Nodes Geometry and Power Allocation in Space-Time Coded Cooperative Wireless Systems

Luca Zuari · Andrea Conti · Velio Tralli

Abstract Cooperative communications are effective in improving the performance and extend the coverage of wireless networks. One issue is to find proper methods to allocate cooperative nodes. In this paper we investigate the effects of relay position and power allocation strategy in cooperative communications employing space-time codes (STCs). We consider non-ideal links between source, relay, and destination enabling the analysis of relay allocation problem based on the performance of each link in realistic scenarios. The frame error rate for various channel conditions, available diversity, relay positions, and transmitted power levels is obtained. Both the situation of balanced and unbalanced transmit power levels for source, relay, and destination are compared. Cooperative pragmatic STCs in block fading channel (BFC) are considered for our analysis. The results provide insight on how to allocate relay nodes based on geometry, link quality, and transmitted power considerations.

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L. Zuari · A. Conti (✉) · V. Tralli
ENDIF and CNIT, University of Ferrara,
Via Saragat 1, 44100 Ferrara, Italy
e-mail: a.conti@ieee.org

L. Zuari
e-mail: lzuari@libero.it

V. Tralli
e-mail: vtralli@ieee.org

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1 Introduction

Cooperative communications are gaining increasing interest as a new communication paradigm involving both transmission and distributed processing which promises significant increase of capacity and diversity gain in wireless networks, by counteracting fading channels with cooperative diversity.

Several issues arise with cooperative diversity schemes such as, among others, channel modeling and implementation aspects [14, 15], protocols and resource management [9], the choice of proper relays [11], power allocation among cooperating nodes [12], and cooperative/distributed STCs [7, 16].

In addition to physical antenna arrays, the relay channel model enables the exploitation of distributed antennas belonging to multiple relaying terminals. This form of space diversity is referred to as cooperative diversity because terminals share antennas and other resources to create a virtual array through distributed transmission and signal processing [1, 10].

With the introduction of STCs it has been shown how, with the use of proper trellis codes, multiple transmitting antennas can be exploited to improve system performance obtaining both diversity and coding gain, without sacrificing spectral efficiency [19]. In Chiani et al. [2] a pragmatic approach to STCs, called pragmatic space-time codes (P-STCs), has been proposed: it simplifies the encoder and decoder structures and also allows a feasible method to search for good codes in

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BFC [20] (P-STC consists in the use of standard convolutional encoders and Viterbi decoders over multiple transmitting and receiving antennas).

In Conti et al. [4] a design methodology of P-STCs for relay networks was provided, resulting in increased flexibility with respect to the above issues. The channel between each transmitting and receiving antenna pair is modeled as BFC which includes a variety of fading rates, from fast fading (i.e., ideal symbol interleaving) to quasi-static [13].

Typically, the link between relay and destination is assumed as an independent, identically distributed (i.i.d.) version of the link between source and destination just adding diversity advantage. In our framework presented here we look carefully on the quality of the links involving the relay (both source–to–relay and relay–to–destination) with two-folds goal: (1) evaluate the performance in a more realistic scenario where also the relay's position impacts the effectiveness of cooperation and the performance at the destination, and (2) provide some insight on how to choose relay nodes based on both geometrical and link quality conditions, and power utilization. The latter point is relevant when power consumption at each node or interference issues have to be addressed.

The paper is organized as follows: in Section 2 we describe the system model and assumptions for the cooperative scheme and in Section 3 we analyze optimal power allocation strategy based on the outage probability. In Section 4 we describe the cooperative space-time code approach for relay networks. Then in Section 5 we provide results in terms of frame error rate (FER) at the destination for various system's parameters choices, and in Section 6 we give our conclusions.

2 System model

The cooperative scheme is depicted in Fig. 1 and follows time-division channel allocations with orthogonal cooperative diversity transmission [16]. The source S divides the time-slot in two equal segments, the first from time t_1 to $t_1 + \Theta$ and the second from $t_2 = t_1 + \Theta$ to $t_2 + \Theta$, where Θ is the segment duration. In the first segment the source broadcasts its coded symbols, in the second segment all the active relays (which are able to decode the message) forward the information through proper encoding to take advantage of the total available diversity. Thus, the design of proper STCs for the two phases is crucial to maximize both achievable diversity and coding gain.

We assume n transmitting antennas at each terminal, m_r and m_{D} receiving antennas at the relay r and at the destination, respectively. Hence, $n_1 = n$ transmitting antennas will be used in the first phase and a total of $n_2 = Rn$ transmitting antennas will be used in the second phase, where R is the number of potential relays. The relay is initialized at the beginning of a data communication session and is kept unchanged over the session. We also assume, to simplify signaling, that the source does not know whether the transmission to relays is successful or not, hence it does not transmit in the second time slot.

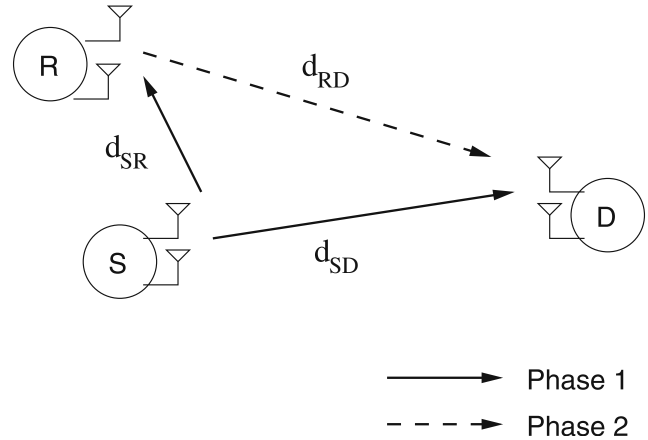


Fig. 1 Two-phase relaying scheme: phase 1 (*continuous line*), phase 2 (*dashed line*). Source, relays and destination nodes are denoted with S, R, D, respectively

We indicate with $c_{r,i}^{(t)}$ the modulation symbol transmitted by relay r ($0 \leq r \leq R$, and $r = 0$ is the source) on the antenna i at discrete time t (i.e. at the tth instant of the encoder clock). Each symbol is assumed to have unit norm and to be generated according to the modulation format by suitable mapping.[1] Note that symbol $c_{0,i}^{(t)}$ is transmitted at time $t_1 + t$, while symbols $c_{r,i}^{(t)}$ for $r > 0$ are transmitted at time $t_2 + t$. In the first phase, symbols $c_{0,i}^{(t)}$ are received by each relay; if decoding is successful,[2] then, in the second phase, the relay re-encodes and forwards the symbols to destination. The received signals corresponding to all symbols $c_{r,i}^{(t)}$ are jointly decoded by the destination at the reference time t. We also denote with $\mathbf{C}^{(t)}$ a super-symbol, which is the vector of the $(R+1)n$ outputs of the "virtual encoder" constituted by source and relays encoders. A codeword is a sequence $\underline{c} = \left(\mathbf{C}^{(1)}, \ldots, \mathbf{C}^{(N)}\right)$ of N super-symbols generated by the source and relays' encoders. This codeword $\underline{c}$ is interleaved before transmission to obtain

[1] Different linear modulation mappings are analyzed in [6] with reference to the properties of the error probability functions.

[2] We assume CRC perfectly recognizing if a codeword is correctly decoded.

the sequence $\underline{c}_I = \mathcal{I}(\underline{c})$, where $\mathcal{I}(\cdot)$ is the interleaving function which orders the sequence such that $\mathbf{C}^{(t)}$ is transmitted at time σ_t. Note that with this notation the permutation is the same for all the transmitting terminals in the two phases.

The channel model includes additive white Gaussian noise (AWGN) and multiplicative flat fading, with Rayleigh distributed amplitudes assumed constant over blocks of B consecutive transmitted space-time symbols and independent from block to block [13]. Perfect channel state information is assumed at the decoder of each node,[3] whereas the transmitters only have to know the mean channel gains for power allocation.

For the destination D the transmitted super-symbol at time σ_t goes through a compound channel described by the $(n_1+n_2)\times m_{\mathrm{D}}$ channel matrix $\mathbf{H}^{(\sigma_t,\mathrm{D})} = [H_0^{(\sigma_t,\mathrm{D})},\ldots,H_R^{(\sigma_t,\mathrm{D})}]^T$ where $H_r^{(\sigma_t,\mathrm{D})} = \left\{h_{r,i,s}^{(\sigma_t,\mathrm{D})}\right\}$, and $h_{r,i,s}^{(\sigma_t,\mathrm{D})}$ is the channel gain between transmitting antenna i $(i=1,\ldots,n)$ of the terminal r and receiving antenna s at the destination D $(s=1,\ldots,m_{\mathrm{D}})$ at time σ_t.

In the BFC model these channel matrices do not change for B consecutive transmissions, hence we actually have only $L = N/B$ possible distinct channel matrix instances per codeword.[4] Denoting by $\mathcal{Z} = \{\mathbf{Z}_1,\ldots,\mathbf{Z}_L\}$ the set of L channel instances, we have $\mathbf{H}^{(\sigma_t,\mathrm{D})} = \mathbf{Z}_l$ for $\sigma_t = (l-1)B+1,...,lB$ and $l=1,\ldots,L$. When the fading block length, B, is equal to one, we have the ideally interleaved fading channel (i.e., independent fading levels from symbol to symbol), while for $L=1$ we have the quasi-static fading channel (fading level constant over a codeword); by varying L we can describe channels with different correlation degrees [13].

Similarly, in the first phase, the rth relay R_r experiences a channel described by the $(n_1\times m_r)$ channel matrix $H_0^{(\sigma_t,\mathrm{R}_r)} = \left\{h_{i,s}^{(\sigma_t,\mathrm{R}_r)}\right\}$ where $h_{i,s}^{(\sigma_t,\mathrm{R}_r)}$ is the channel gain between transmitting antenna i $(i=1,\ldots,n)$ of the source and receiving antenna s at the relay R_r $(s=1,\ldots,m_r)$ at time σ_t.

At the destination the sequence of received signal vectors, after the deinterleaving, is $\left(\mathbf{R}^{(1,\mathrm{D})},\ldots,\mathbf{R}^{(N,\mathrm{D})}\right)$, where the received vector at time t is $\mathbf{R}^{(t,\mathrm{D})} = \left[r_1^{(t,\mathrm{D},1)}r_1^{(t,\mathrm{D},2)}\cdots\ r_{m_{\mathrm{D}}}^{(t,\mathrm{D},1)}r_{m_{\mathrm{D}}}^{(t,\mathrm{D},2)}\right]^T$ with

$$r_s^{(t,\mathrm{D},1)} = \sqrt{E_0}\sum_{i=1}^{n} h_{0,i,s}^{(\sigma_t,\mathrm{D})} c_{0,i}^{(t)} + \eta_s^{(t,\mathrm{D},1)} \tag{1}$$

in the first phase $(s=1,\ldots,m_{\mathrm{D}})$ and

$$r_s^{(t,\mathrm{D},2)} = \sum_{r=1}^{R}\sqrt{E_r}\sum_{i=1}^{n} h_{r,i,s}^{(\sigma_t,\mathrm{D})} c_{r,i}^{(t)} + \eta_s^{(t,\mathrm{D},2)} \tag{2}$$

for the second phase. In this equation $r_s^{(t,\mathrm{D},l)}$ is the signal-space representation of the signal received by antenna s at time t in phase l, the noise terms $\eta_s^{(t,\mathrm{D},l)}$ are i.i.d. complex Gaussian random variables (r.v.s), with zero mean and variance $N_0/2$ per dimension, and the r.v.s $h_{r,i,s}^{(\sigma_t,D)}$ represent the de-interleaved complex Gaussian fading coefficients. Similarly, the received signal vector at the rth relay in phase 1 at time t is $\mathbf{R}^{(t,\mathrm{R}_r)} = \left[r_1^{(t,\mathrm{R}_r)}\cdots r_{m_r}^{(t,\mathrm{R}_r)}\right]^T$ with components

$$r_s^{(t,\mathrm{R}_r)} = \sqrt{E_0}\sum_{i=1}^{n} h_{i,s}^{(\sigma_t,\mathrm{R}_r)} c_{0,i}^{(t)} + \eta_s^{(t,\mathrm{R}_r,1)}, \quad s=1,\ldots,m_{\mathrm{R}_r}\,. \tag{3}$$

We assume spatially uncorrelated channels with elements $h_{r,i,s}^{(t,\mathrm{D})}$ and $h_{i,s}^{(t,\mathrm{R}_r)}$ independent, non-identically distributed (i.n.i.d.) Complex Gaussian r.v.s with zero mean and variance per dimension given by

$$\begin{cases} \Delta_{\mathrm{SD}}/2 & \text{for } h_{0,i,s}^{(t,\mathrm{D})}, \\ \Delta_{\mathrm{R}_r\mathrm{D}}/2 & \text{for } h_{r,i,s}^{(t,\mathrm{D})}, \\ \Delta_{\mathrm{SR}_r}/2 & \text{for } h_{i,s}^{(t,\mathrm{R}_r)}, \end{cases} \tag{4}$$

where, if we normalize all the distances to source-destination distance d_{SD}, we have $\Delta_{\mathrm{SD}} = 1$ and

$$\Delta_{\mathrm{SR}_r} = \left(d_{\mathrm{SR}_r}/d_{\mathrm{SD}}\right)^{-\beta}$$
$$\Delta_{\mathrm{R}_r\mathrm{D}} = \left(d_{\mathrm{R}_r\mathrm{D}}/d_{\mathrm{SD}}\right)^{-\beta}\,.$$

Here, d_{SR_r} is the distance between source and relay R_r, and $d_{\mathrm{R}_r\mathrm{D}}$ is the distance between relay R_r and destination; at the distance d a path-loss proportional to d^β is assumed.

The average transmitted energy per symbol E_{s}, when all relays are active, is equal to

$$E_{\mathrm{s}} = \sum_{r=0}^{R} E_r/(R+1)\,. \tag{5}$$

The energy transmitted per information bit is $E_{\mathrm{b}} = E_{\mathrm{s}}/(\nu R_{\mathrm{c}})$ where ν is the number of bits per modulation symbol and R_{c} is the code-rate of the cooperative space-time code.

[3] As well known, nonperfect CSI at the receiver leads to some performance degradation, but this is not within the scope of paper investigation.

[4] For the sake of simplicity we assume N and B such that L is an integer.

As far as power allocation among source and relays is concerned (i.e., the values of E_r) we consider three different strategies:

- *Uniform power allocation*: the source and all relays transmit with equal power, thus $E_r = E_s$ for $r = 0, 1, \ldots, R$;
- *Ideal power control*: the power among source and relays are balanced such that the average received power at the destination is the same. Thus, the source transmits with E_0 and the rth relay with $E_r = E_0 \Delta_{SD} / \Delta_{R_r D}$ where

$$E_0 = E_s (R+1) / \left(1 + \sum_{r=1}^{R} 1/\Delta_{R_r D}\right). \tag{6}$$

- *Outage-optimal power allocation*: the power is distributed among source and relays such that the outage probability at the destination is minimized. The analysis for this strategy is given in Section 3.

3 Outage-optimal power allocation

The outage-optimal power allocation technique consists in allocating the power among the source and the relays such that the outage probability[5] at the destination is minimized. Here, we extend the approach in Hasna and Alouini [8] to multiple input multiple output (MIMO) links between each pair of nodes. We aim to capture the role of diversity for a system with multiple MIMO links by means of a tractable model. We consider that each receiver combines signals over multiple antennas through maximal ratio diversity processing and that the destination selects the signal with greater signal-to-noise ratio (SNR), among the one from the source or those from the relay. This enables us to consider the power allocation between source and relay independently of the particular space-time cooperative scheme.

We restrict here our derivation to the case with one relay ($R = 1$). The source transmits with energy per symbol $x_S E_s$ and the relay forwards the message to the destination with energy per symbol $x_R E_s$ if it is not in outage (i.e, the error probability at the relay is lower than a target error probability and, consequently, the SNR is greater than the required SNR γ_{th}). The constraint on the total energy per symbol over the two phases is given by $x_S,\ x_R \in [0, x_t]$ and $x_S + x_R = x_t = 2$.

The selection diversity between signals from source and relay gives the following outage probability at the destination

$$\begin{aligned} P_{out} &= \left[1 - \mathbb{P}\{\gamma_{SR} \geq \gamma_{th}\}\, \mathbb{P}\{\gamma_{RD} \geq \gamma_{th}\}\right] \mathbb{P}\{\gamma_{SD} < \gamma_{th}\} \\ &= \left[1 - (1 - F_{SR}(\gamma_{th}))(1 - F_{RD}(\gamma_{th}))\right] F_{SD}(\gamma_{th}) \end{aligned} \tag{7}$$

where the cumulative distribution function (c.d.f.) of the SNR $\gamma_{\mathbb{L}}$ on the link $\mathbb{L}$ (the reader may substitute SD, SR, or RD to $\mathbb{L}$ depending on the link considered) after selection is

$$F_{\mathbb{L}}(z) = 1 - e^{z/\overline{\gamma}_{\mathbb{L}}} \sum_{d=0}^{D_{\mathbb{L}}-1} \frac{1}{d!} \left(\frac{z}{\overline{\gamma}_{\mathbb{L}}}\right)^d. \tag{8}$$

which depends on the diversity $D_{\mathbb{L}}$ and mean SNR $\overline{\gamma}_{\mathbb{L}}$. By considering the channel model assumptions made in the previous section, we have: $\overline{\gamma}_{SD} = x_S \Delta_{SD} \overline{\gamma}$, $\overline{\gamma}_{SR} = x_S \Delta_{SR} \overline{\gamma}$, and $\overline{\gamma}_{RD} = x_R \Delta_{RD} \overline{\gamma}$, where $\overline{\gamma}$ is the SNR E_s/N_0.

We now evaluate the power partition $\mathbf{x} = (x_S,\ x_R)$ which minimizes the outage probability in Eq. 7, i.e

$$\begin{aligned} &\min_{\mathbf{x}} P_{out} \\ &\text{s.t.}\ \ x_S + x_R = x_t, \quad x_S, x_R \geq 0 \end{aligned} \tag{9}$$

Since P_{out} is continuous and differentiable in x_S and x_R when $0 < x_S, x_R < x_t$, the solutions can be found among[6] the zeros of the first derivative of P_{out} with respect to x_S, after the substitution $x_R = x_t - x_S$, i.e

$$\begin{aligned} &\left[F'_{SR} \Delta_{SR} \overline{\gamma} (1 - F_{RD}) - F'_{RD} \Delta_{RD} \overline{\gamma} (1 - F_{SR})\right] F_{SD} \\ &\quad + [1 - (1 - F_{SR})(1 - F_{RD})]\, F'_{SD} \Delta_{SD} \overline{\gamma} = 0 \end{aligned} \tag{10}$$

where $F_{\mathbb{L}} = F_{\mathbb{L}}(\gamma_{th})$ and

$$F'_{\mathbb{L}} = \frac{dF_{\mathbb{L}}(\gamma_{th})}{d\overline{\gamma}_{\mathbb{L}}} = -e^{-\gamma_{th}/\overline{\gamma}_{\mathbb{L}}} \frac{1}{\overline{\gamma}_{\mathbb{L}}} \frac{1}{(D_{\mathbb{L}} - 1)!} \left(\frac{\gamma_{th}}{\overline{\gamma}_{\mathbb{L}}}\right)^{D_{\mathbb{L}}}. \tag{11}$$

Equation 10 can be solved numerically to obtain the power allocation between source and relay. To gain

[5] The concept of outage probability in wireless communications is well known. For application to digital communications with a given target error probability see, e.g., [3, 5].

[6] And $x_S = x_t$ if no useful solution is found.

insights on the solution one can consider the approximation for large SNRs. Then Eqs. 8 and 11 result in

$$F_{\mathbb{L}} \approx \frac{1}{D_{\mathbb{L}}!}\left(\frac{\gamma_{\text{th}}}{\overline{\gamma}_{\mathbb{L}}}\right)^{D_{\mathbb{L}}}$$

$$F'_{\mathbb{L}} \approx -\frac{1}{\overline{\gamma}_{\mathbb{L}}(D_{\mathbb{L}}-1)!}\left(\frac{\gamma_{\text{th}}}{\overline{\gamma}_{\mathbb{L}}}\right)^{D_{\mathbb{L}}}$$

By considering $D_{\mathbb{L}} = D$ for all links, and taking the limit for large $\overline{\gamma}$ (asymptotic region), we obtain the solution

$$\left(\frac{x_{\text{S}}}{x_{\text{R}}}\right)^{D}\left(\frac{x_{\text{S}}}{x_{\text{R}}}-1\right) = 2\left(\frac{\Delta_{\text{RD}}}{\Delta_{\text{SR}}}\right)^{D} . \tag{12}$$

For $D = 1$, the admissible solution for Eq. 12 results in the closed form

$$\frac{x_{\text{S}}}{x_{\text{R}}} = 0.5\left(1+\sqrt{1+8\Delta_{\text{RD}}/\Delta_{\text{SR}}}\right) . \tag{13}$$

Note that the solution (Eq. 13) is in the same form of the result obtained in Su et al. [18] where a power allocation technique has been presented to minimize the symbol error probability at the destination for uncoded systems with $D = 1$; the constant 8 is replaced by a modulation dependent constant. This indicates that outage-optimization has the same structure of error probability-optimization. This motivates the use of outage-optimization to easily derive power allocation solutions.

For $\Delta_{\text{RD}}/\Delta_{\text{SR}}$ approaching zero (relay very close to the source) the solution $x_{\text{S}}/x_{\text{R}}$ tends to 1 (uniform power allocation), while for $\Delta_{\text{RD}}/\Delta_{\text{SR}}$ approaching infinity (relay very close to the destination) the solution $x_{\text{S}}/x_{\text{R}}$ tends to $1/(D+1) + 2^{1/(D+1)}\left(\Delta_{\text{RD}}/\Delta_{\text{SR}}\right)^{D/(D+1)}$. A possible approximated solution is thus given by

$$x_{\text{S}}/x_{\text{R}} = \max\left\{1, \frac{1}{D+1} + 2^{\frac{1}{D+1}}\left(\Delta_{\text{RD}}/\Delta_{\text{SR}}\right)^{\frac{D}{D+1}}\right\} .$$

In Fig. 2 we illustrate the different solutions for the ratio $x_{\text{S}}/x_{\text{R}}$ as function of $\Delta_{\text{RD}}/\Delta_{\text{SR}}$ with various conditions of link diversity and protection ratio $\overline{\gamma}/\gamma_{\text{th}}$. We first note the different behaviors of the two scenarios, one with balanced link diversity and the other with unbalanced link diversity. In the first case the approximated solution for $D = 2$ appears quite accurate. In the second case the sensitivity to protection ratio increases leading to a noticeable difference between the solutions with small and large (or asymptotic) SNR.

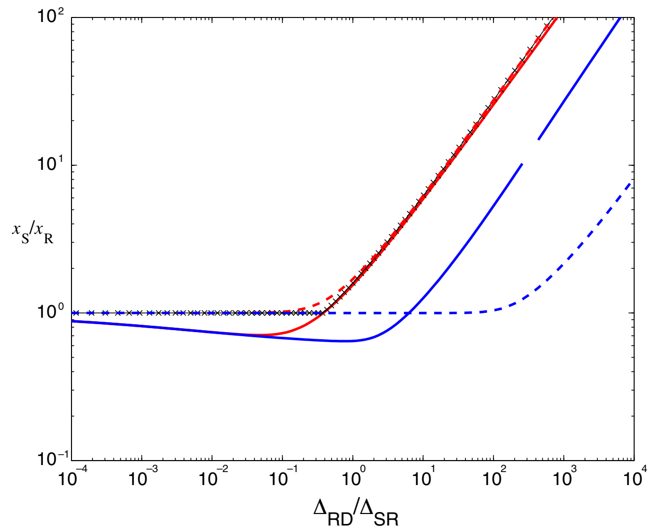


Fig. 2 Power ratio $x_{\text{S}}/x_{\text{R}}$ as function of $\Delta_{\text{RD}}/\Delta_{\text{SR}}$. *Red lines* refer to system with $D_{\text{SD}} = D_{\text{SR}} = D_{\text{RD}} = 2$, *blue lines* to system with $D_{\text{SD}} = D_{\text{RD}} = 2$, $D_{\text{SR}} = 4$. *Solid lines* refer to $\overline{\gamma}/\gamma_{\text{th}} = 0$ dB, *dashed lines* to $\overline{\gamma}/\gamma_{\text{th}} = 20$ dB. *Crosses* refer to approximated solution for $D = 2$

4 Cooperative space-time code for relaying

In the case of the two-phase relaying scheme shown in Fig. 1, the probability of transmission failure over the two phases depends on the number of relays available for cooperation and on the quality of links source-destination, source-relays, and relays-destination.

Depending on terminals' positions the relays are set by looking at those that are able to guarantee effective cooperation with the source and to satisfy the target performance at the destination. Sometimes, due to fading, it may happen that a relay is not able to decode the source codewords in the first phase. Let us denote by $P_{\text{e}}^{(\text{SD})}$ the error probability for source-destination link, $P_{\text{e}}^{(\text{SR}_r)}$ the error probability for the source- rth relay link, and with $P_{\text{e}}^{(\text{S}\mathcal{R}\text{D})}$ the error probability for the link from the source plus active relays ($\mathcal{R}$ being the set of active relays) to the destination. Note that these performance metrics are functions of system parameters as

$$P_{\text{e}}^{(\text{SD})} = f^{(\text{SD})}\left(E_{\text{s}}, N_0\right)$$

$$P_{\text{e}}^{(\text{SR}_r)} = f^{(\text{SR}_r)}\left(E_{\text{s}}, \Delta_{\text{SR}_r}, N_0\right)$$

$$P_{\text{e}}^{(\text{S}\mathcal{R}\text{D})} = f^{(\text{S}\mathcal{R}\text{D})}\left(E_{\text{s}}, \left\{\Delta_{\text{R}_r\text{D}}\right\}, N_0\right) .$$

The error probability at destination for the case of one relay is given by

$$P_e = P_e^{(\text{SR}_1)} P_e^{(\text{SD})} + \left(1 - P_e^{(\text{SR}_1)}\right) P_e^{(\text{SR}_1\text{D})} . \tag{14}$$

This can be generalized for multiple relays. As an example, for two relays it results

$$P_e = P_e^{(\mathrm{SR}_1)} P_e^{(\mathrm{SR}_2)} P_e^{(\mathrm{SD})} + \left(1 - P_e^{(\mathrm{SR}_1)}\right) P_e^{(\mathrm{SR}_2)} P_e^{(\mathrm{SR}_1\mathrm{D})} + P_e^{(\mathrm{SR}_1)} \left(1 - P_e^{(\mathrm{SR}_2)}\right) P_e^{(\mathrm{SR}_2\mathrm{D})} + \left(1 - P_e^{(\mathrm{SR}_1)}\right) \left(1 - P_e^{(\mathrm{SR}_2)}\right) P_e^{(\mathrm{SR}_1\mathrm{R}_2\mathrm{D})}. \quad (15)$$

For the goal of our paper, we consider space-time trellis codes for relaying networks by using the pragmatic approach of Chiani et al. [2] and Tralli et al. [20]. The *pragmatic* approach uses a low-complexity architecture for STC where the code components are built by the concatenation of a binary convolutional encoder and binary phase shift keying (BPSK) or quaternary phase shift keying (QPSK) modulator. The k information bits are encoded by a convolutional encoder with rate $k/(n\nu)$ and the $n\nu$ output bits are divided into n streams, one for each transmitting antenna, of BPSK ($\nu = 1$) or QPSK ($\nu = 2$) symbols that are obtained from a natural (Gray) mapping of ν bits. Then, each stream of symbols is eventually interleaved. If μ is the encoder constraint length then the associated trellis has $2^{k(\mu-1)}$ states.

We can describe a P-STC for cooperative communication, obtained by joining the $R+1$ code components of the cooperating transmitters, by using the trellis of each encoder (the same as for the space-time code CC) labelling the generic branch from state S_i to state S_j with the super-symbol $\widetilde{\mathbf{C}}_{S_i \to S_j} = [\widetilde{c}_{0,1}, \ldots, \widetilde{c}_{R,n}]^T$. For BPSK the symbol $\widetilde{c}_{r,i}$ is the output (in antipodal form) of the ith generator of the rth transmitter. One of the advantages of the pragmatic architecture is that the maximum likelihood (ML) decoder is the usual Viterbi decoder for the convolutional encoder adopted (same trellis), with a simple modification of the branch metrics. Being $\{\widetilde{c}_{r,i}\}$ the set of output symbols labelling the branch, the branch metric for the Viterbi decoder is

$$\sum_{s=1}^{m_\mathrm{D}} \left(\left| r_s^{(t,\mathrm{D},1)} - \sqrt{E_0} \sum_{i=1}^{n} h_{0,i,s}^{(\sigma_t,\mathrm{D})} \widetilde{c}_{0,i} \right|^2 + \left| r_s^{(t,\mathrm{D},2)} - \sum_{r=1}^{R} \sqrt{E_r} \sum_{i=1}^{n} h_{r,i,s}^{(\sigma_t,\mathrm{D})} \widetilde{c}_{r,i} \right|^2 \right). \quad (16)$$

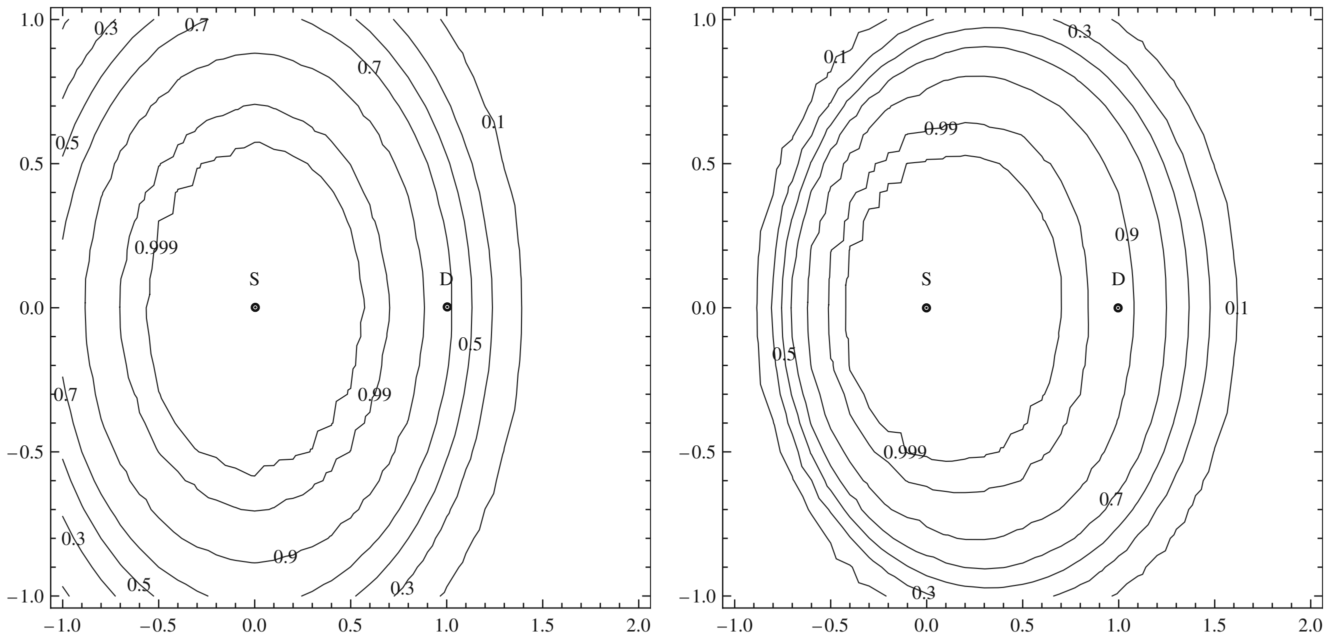
In Conti et al. [4] it was discussed how to perform an efficient search for generators of cooperative P-STCs in BFC. For the design of the coding scheme with cooperative relays it is generally recognized that the code components used by the source in phase 1 should maximize diversity and coding gain for each link connecting the source to relays and destination. The other code components should be designed to maximize diversity and coding gain of the entire cooperative code, that is the code including all the code components transmitted during phase 1 and 2, for any possible number of cooperative relays [17].

By assuming that the cooperative code is obtained by joining code components in phase 2 from every relay able to decode the source message, the code may be designed as STC with overlay construction. For P-STCs this gives cooperative overlay pragmatic space-time codes (COP-STCs). With this method, a good code for R relays is designed starting from a good code for $R-1$ relays and by adding the remaining best code component that maximize diversity and coding gain of the final code. In this way the first code component used by the source in phase 1 is always a good code. In the case of a fixed set of more than one cooperating relays the sequence of pre-designed additional code components can be assigned to the relays ranked in order of average link quality, so that they are used with higher probability in the same order for which they have been designed.

5 Numerical results and relay allocation

The performance figure we are interested in are the effective relay cooperation and, most important, the mean FER at the destination averaged over rapid processes, such those related to BFC. Here, they are evaluated for COP-STC as a function of SNR, relay location, BFC characteristics, and power allocation strategy. We first discuss the effect on relay allocation. We refer to a scenario in which a source transmits to a destination with one potential relay (e.g., this is the situation for the downlink of cellular systems where the source is the base-station, the destination is the mobile-station, and the relay is a terminal enabled to act as a cooperating node placed in any position over the space).

The results are obtained for BPSK constellation signaling, COP-STC with 8 states, code-rate 1/4, generators $(13, 15, 11, 17)_8$ as from Conti et al. [4], $N = 130$, $n = 2$ transmitting antennas per node, $m_\mathrm{R} = 2$ receiving antennas at the relay, $m_\mathrm{D} = 1$ receiving antenna at the destination, and with the three power allocation strategies discussed in Section 2. Various BFCs (i.e., values of L) are considered. The SNR is defined as

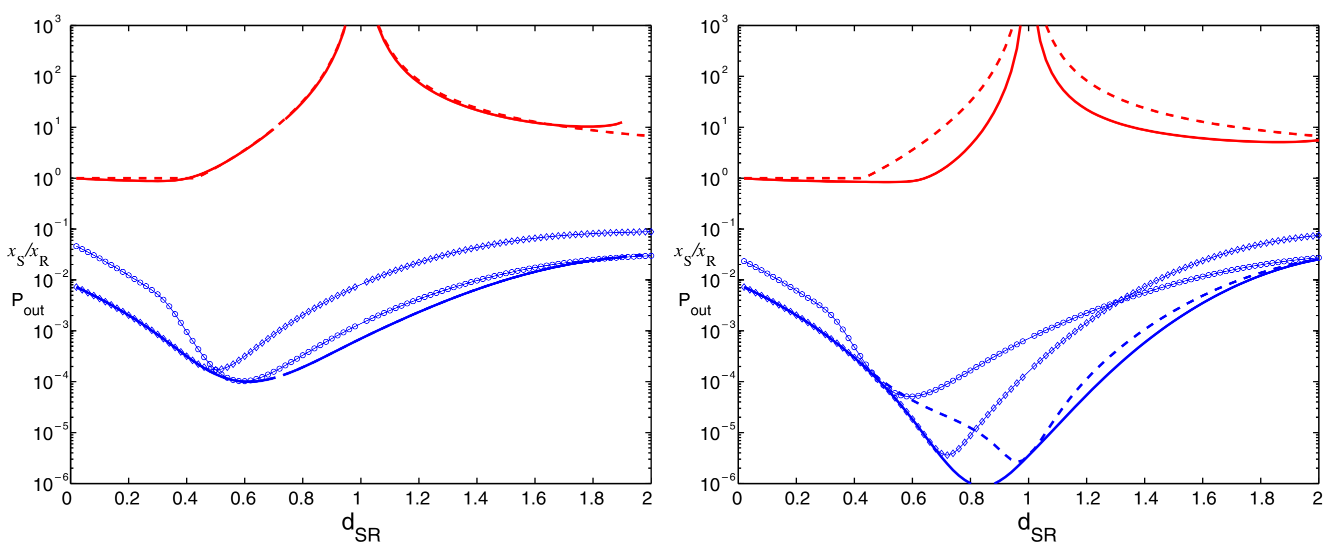


(a) P_{coop} contours for $L = 1$ with uniform power allocation.

(b) P_{coop} contours for $L = 1$ with ideal power control.

Fig. 3 Spatial distribution of P_{coop} depending on relay's position. BFC for E_b/N_0 =5 dB

E_b/N_0 per receiving antenna element where, for a fair comparison among situations with different number of relays, E_b is the total energy per information bit over all transmitting nodes and averaged with respect to fading. All possible relay's positions on a bi-dimensional scaled plane are evaluated with distances normalized to d_{SD} (source in coordinates (0,0) and destination in (1,0)). Then, in the last figure we also consider the case of destination's positions varying and with fixed relay in coordinates (1,0) with distances normalized to d_{SR}. The path-loss coefficient is $\beta = 3.5$ which is a typical choice for many wireless scenarios.



(a) $D_{SD} = D_{SR} = D_{RD} = 2$ (balanced).

(b) $D_{SD} = D_{RD} = 2$, $D_{SR} = 4$ (unbalanced).

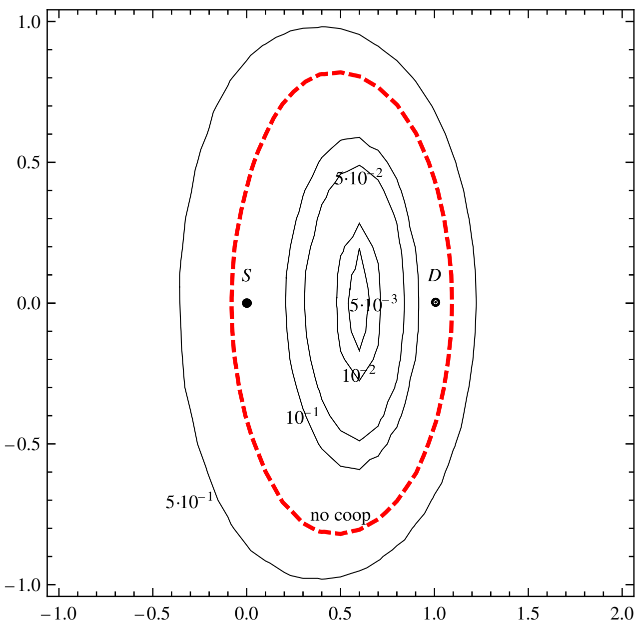
Fig. 4 Power ratio x_S/x_R and outage probability as function of relay position d_{SR_r} in a linear scenario with $d_{SD} = 1$, $\overline{\gamma}/\gamma_{th} = 3$ dB. *Solid lines* refer to optimal power allocation, *dashed lines* to optimal power allocation with asymptotic solution, *line with circles* to ideal power control, *line with diamonds* to uniform power allocation

5.1 Spatial distribution of the cooperation probability

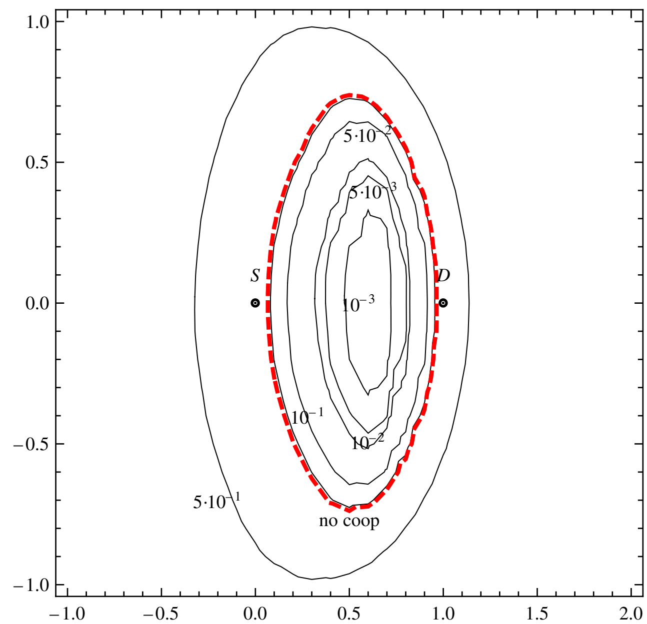
Figure 3 shows the probability of relay's cooperation P_{coop} for various relay's positions with two power allocation strategies: uniform and ideal power control. It is defined as $P_{\text{coop}} = 1 - P_e^{(\text{SR})}$. As expected the cooperation probability is large when the relay is placed close to the source. When ideal power control is adopted the regions with high probability of cooperation move toward the destination, and the same is expected for the case of optimal power allocation. Of course, the spatial distribution of the cooperation probability is only one aspect; in fact the cooperative setting aims at minimizing the FER at the destination. Situations in which the relay always cooperates but it has a very bad link quality with the destination, as well as situations where the relay would really improve the performance but, due to its bad link to the source, rarely cooperates, have to be avoided.

5.2 Outage probability

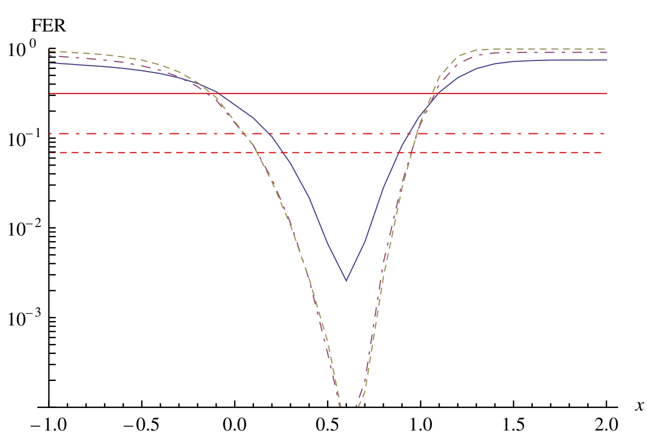
Before showing the performance in terms of FER we briefly introduce and discuss the behavior of the



(a) FER contours for $L = 1$.

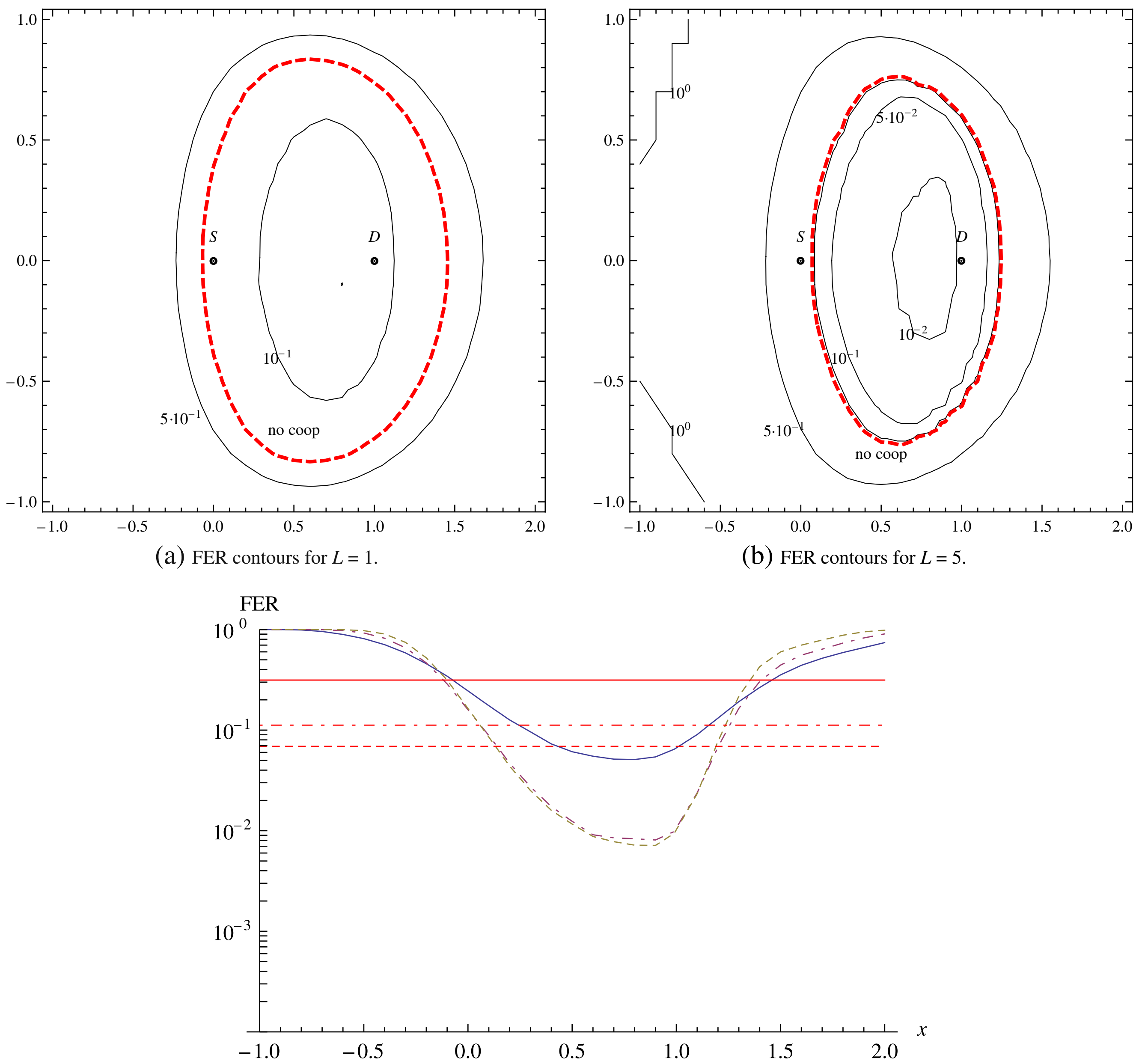


(b) FER contours for $L = 5$.



(c) FER along the line containing S and D for $L = 1, 5, 130$ (continuous, dotted, dashed).

Fig. 5 FER with uniform power allocation for various relay's positions and $E_b/N_0 = 5$ dB. *Red lines* refer to the case without cooperation different power allocation strategies in term of outage probability. Figure 4 shows the values of x_S/x_R and outage probability as a function of relay position d_{SR} in a one-dimensional scenario (relay positioned on the line containing source and destination) with $d_{SD} = 1$. The two cases of balanced and unbalanced link diversity are also compared; in the last case, the source-relay link is made more reliable by increasing its diversity. We note that optimal power allocation increases the power of the source when the relay is close to destination. The outage probability has a minimum located at $d_{SR} \approx 0.6$; it moves toward the destination as the diversity D_{SR} increases. Note that in the system with balanced link diversity the asymptotic solution is quite reliable to get optimal power allocation. Uniform power allocation is a good strategy when d_{SR} is smaller than d_{SD}; however it is looser when the relay is close to destination. Ideal power control achieves a good performance in the case



(a) FER contours for $L = 1$.

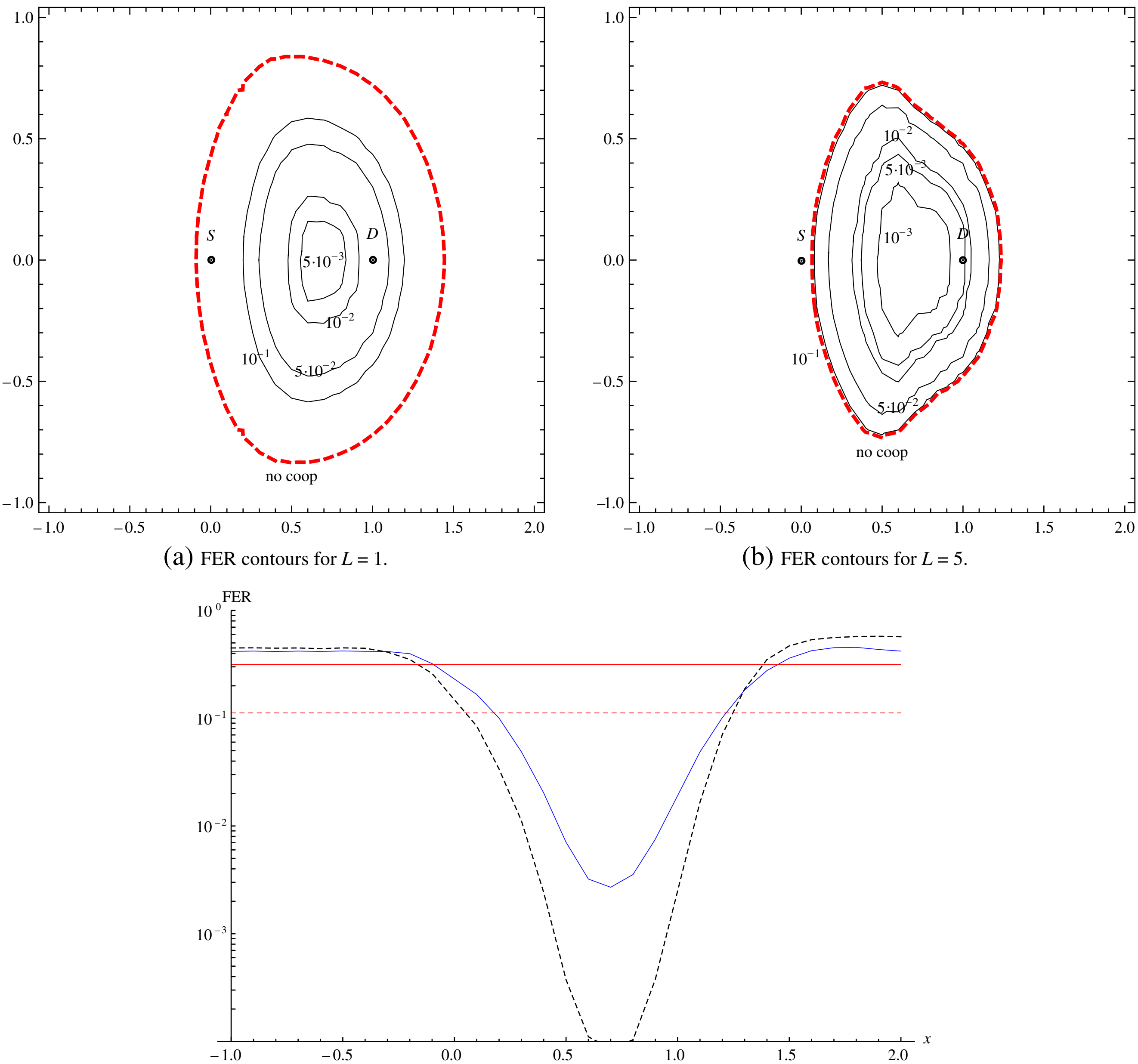
(b) FER contours for $L = 5$.

(c) FER along the line containing S and D for $L = 1, 5, 130$ (continuous, dotted, dashed).

Fig. 6 FER with ideal power control for various relay's positions and $E_b/N_0 = 5$ dB. *Red lines* refer to the case without cooperation of balanced link diversity, whereas it appears poor for the unbalanced diversity case.

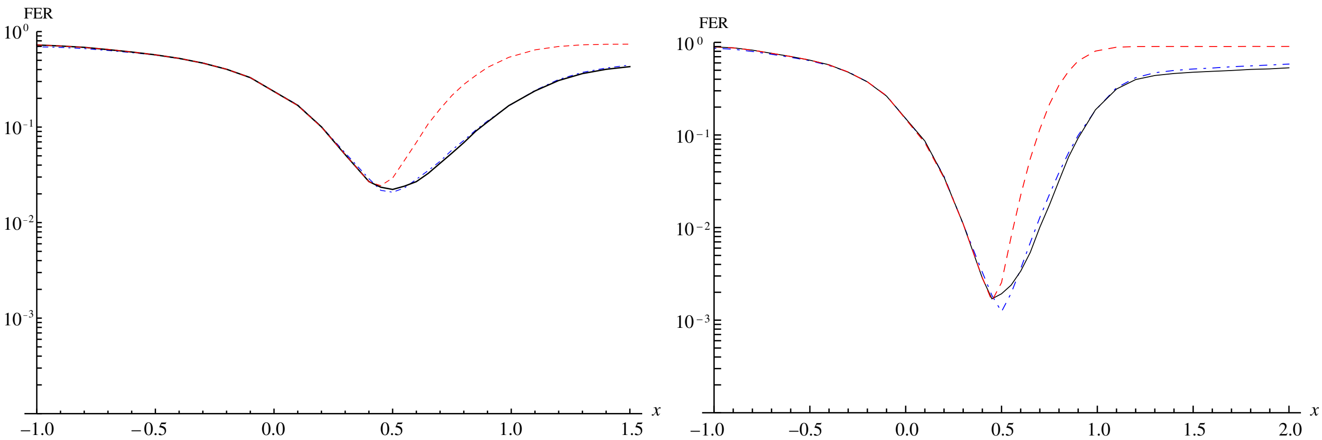
5.3 Frame error rate at the destination

The FER at the destination is obtained by simulating each single performance term of Eq. 14 considering the actual relay's position, SNR, BFC, and power allocation. In Fig. 5 the FER with uniform power allocation is shown for $L = 1, 5, 130$, and $E_b/N_0 = 5$ dB. In red dash the FER for the link source–to–destination in the absence of relaying is shown to understand where the cooperative relay improves the performance. We can note that the region where cooperation is beneficial is large and centered in a point toward the destination. Contours enlarge, as well as the minimum FER decreases, when L increases. Results for $L = 5$ and $L = 130$ are overlapped since the number of states



(c) FER along the line containing S and D for $L = 1, 5$ (continuous, dashed).

Fig. 7 FER with optimal power allocation for various relay's positions and $E_b/N_0 = 5$ dB. *Red lines* refer to the case without cooperation



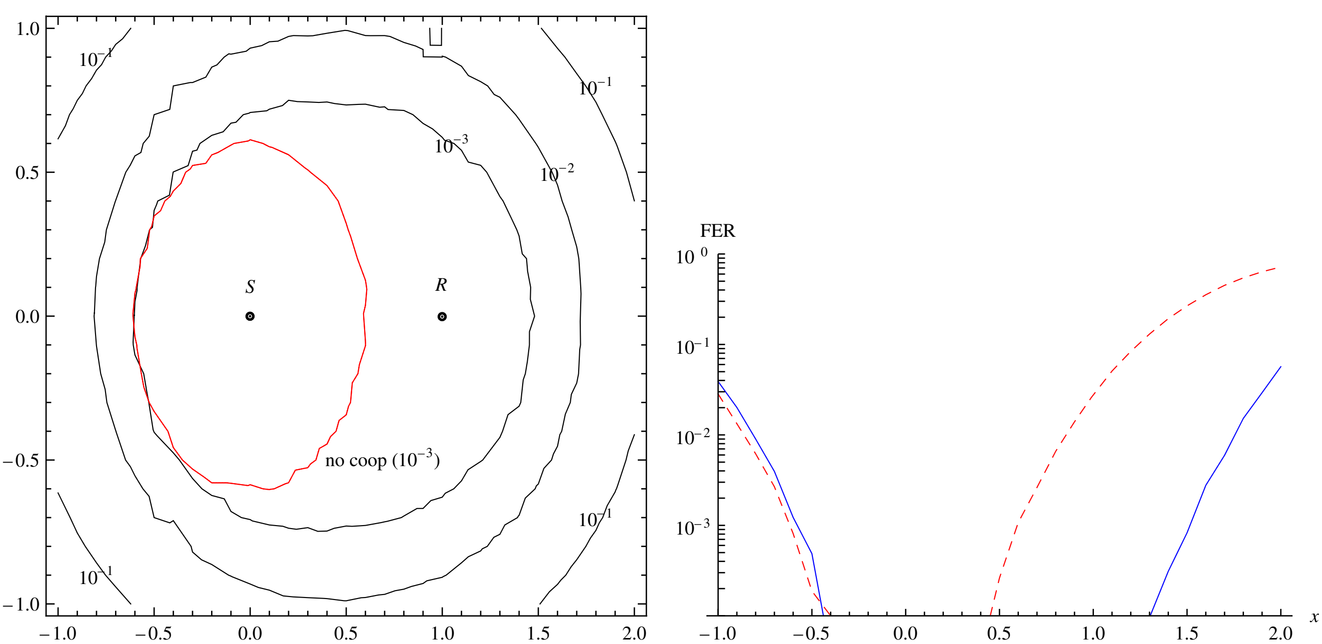
(a) $n = 2, m_R = m_D = 1$ and $L = 1$. (b) $n = 2, m_R = m_D = 1$ and $L = 5$.

Fig. 8 FER with uniform (*dashed*) and outage-optimal power allocation using asymptotic solution with different D (*continuous* for $D = 2$ on the *left* and $D = 4$ on the *right*, *dotted* for $D = 1$) for various relay's positions along the line containing S and D, and for $E_b/N_0 = 5$ dB

of the chosen COP-STC saturates the available time-diversity. The results enable the comparison of the FER for cooperative and non-cooperative settings, and also provide a geometrical view of where the best relay should be located.

Results in the same form but for the case of ideal power control can be seen in Fig. 6. Here, the main difference due to the power allocation strategy is given by the fact that the minimum FER is greater with ideal power control but at the same time the FER is less sensitive to the relay's position.

Figure 7 shows the results for outage-optimal power allocation. It is confirmed that this power allocation strategy provides the best performance. It is interesting



(a) FER contours for $L = 1$. (b) FER along the line containing S and R for $L = 1$.

Fig. 9 FER with optimal power allocation for fixed relay's position in (1, 0) and various destination's positions; $E_b/N_0 = 13$ dB. *Red lines* refer to the case without cooperation to observe that the minimum FER is not as close to destination as for outage probability. Outage optimal power allocation is also evaluated and compared to uniform power allocation in the case of balanced link diversity (in this case $D = 2$ over all links). The results are reported in Fig. 8 which shows that in this case the minimum FER moves to $d_{SR_r} \approx 0.5$. Optimal power allocation has in this case a very small sensitivity to the value of D used to derive the solutions of x_S and x_R.

In Fig. 9 we show the performance for all possible destination's positions on a bi-dimensional scaled plane (source in coordinates (0,0) and relay in (1,0)), evaluated with distances normalized to d_{SR}. The optimal power allocation is considered. This figure give us a clear idea of the benefits of a cooperating relay in terms of coverage increasing.

Remark Finally, it should be underlined that in the case of ideal power control and outage-optimal power allocation the transmitted power decreases as the relay is close to the destination. For the example application discussed above, when the destination is near to the cell edge, this causes a lower level of intercell interference from the relay to others cells. In general, these kind of results enable to understanding power consumption of relay and represent an input for the analysis of the interference caused by relay.

5.4 Relay allocation strategy

The presented results provide insights useful to define methods for relay assignment. The choice of relay's position depends on a proper balancing between the FER at the destination and the transmitted power at the relay. We summarize the following considerations: (1) the relay should be located between source and destination, preferably closer to the destination than to the source when the source-relay link works with a diversity larger than those in the other links; (2) uniform power allocation is good when the relay is nearest to the source than to the destination, whereas ideal power control is acceptable when all the links have balanced diversity; (3) with ideal power control or optimal power allocation the power level transmitted by the relay is lower near the destination. Thus, even if the probability of cooperation increases as the relay is close to the source, the performance metrics indicate that the goodness of the link relay-destination is important and the beneficial regions for cooperation are closer to the destination especially when a power allocation technique is employed.

6 Conclusions

We analyzed relayed communications with cooperative overlay pragmatic space-time codes in block fading channels by considering a real placement of the relay and the quality for the links source–to–destination, source–to–relay, and relay–to–destination. Various power allocation techniques are analyzed: uniform power allocation, ideal power control, and outage-optimal power allocation. This framework enabled considerations on relay allocation criteria based on a balancing between the space-time code FER at the destination and the power level transmitted by the relay depending on its position.

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